CHAPTER FOCUS

The study of Pythagoras' theorem introduces the use of the rule $c^2 = a^2 + b^2$ to determine an unknown length. Students will learn to identify right-angled triangles and solve for unknown lengths using $c^2 = a^2 + b^2$. They will apply this information in many different two-dimensional problems to identify right-angled triangles and find unknown lengths, including the use of surds in exact answers. These skills will be reinforced through practical problem-solving tasks.

Outcomes

Right-Angled Triangles (Pythagoras)
[Stage 4]

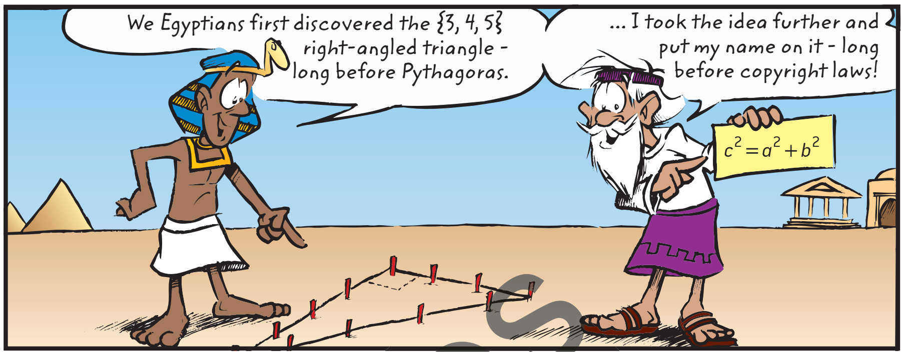
MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols

MA4-2WM applies appropriate mathematical techniques to solve problems

MA4-16MG applies Pythagoras' theorem to calculate side lengths in right-angled triangles, and solves related problems

PYTHAGORAS' THEOREM

3



Contents

Syllabus references (See page x for details.)

Measurement and Geometry, **Number and Algebra**
Selections from ***Right-Angled Triangles (Pythagoras)***, ***Equations*** [Stage 4]

- Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles. (ACMMG222)
- Investigate the concept of irrational numbers. (ACMNA186)
- Solve simple quadratic equations.

Working Mathematically

- Communicating
- Problem Solving
- Reasoning
- Understanding
- Fluency

Key ideas

- Right-angled triangles have one angle of 90°.
- Pythagoras theorem ($c^2 = a^2 + b^2$) can be used to find an unknown side length of right-angled triangles.
- The 3-4-5 rule (Pythagorean triad) can be used to determine other triads.
- Exact distances of side lengths can be determined when using surds (irrational numbers).

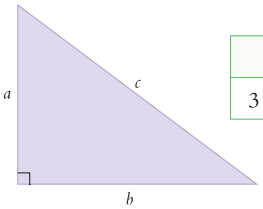
Language

approximately equal to	interval	recurring decimal
decimal places	irrational numbers	right-angled triangles
diagonal	perimeter	square root
exact value	Pythagoras' theorem	surds
hypotenuse	Pythagorean triads	terminating decimal

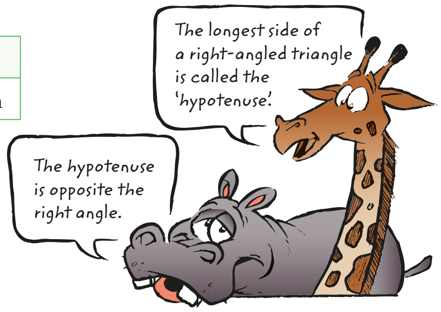
3:01 Investigating right-angled triangles

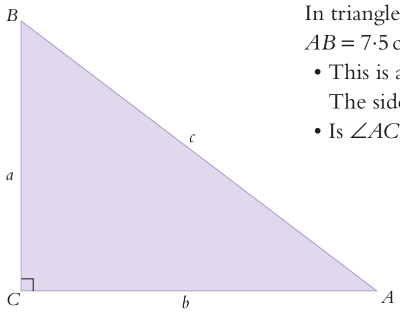
The Egyptians made use of the fact that a triangle that had sides 3 units, 4 units and 5 units in length was always a right-angled triangle. However, around 550 BCE a religious group led by the Greek philosopher Pythagoras discovered the relationship between the sides of all right-angled triangles.

As we investigate right-angled triangles we will call the side lengths *a*, *b* and *c*, where *c* will always stand for the length of the longest side. We call the longest side of a right-angled triangle the **hypotenuse**.



a	*b*	*c*
3 cm	4 cm	5 cm





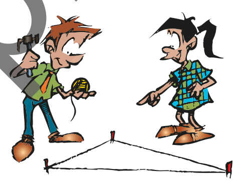
In triangle *ABC* (Δ*ABC*), *BC* = 4·5 cm, *CA* = 6 cm and *AB* = 7·5 cm.

- This is also a {3, 4, 5} triangle if we let each unit be 1·5 cm. The side lengths are: 3 × 1·5 cm, 4 × 1·5 cm and 5 × 1·5 cm.
- Is ∠*ACB* a right angle? Which side is the hypotenuse?

a	*b*	*c*
4·5 cm	6 cm	7·5 cm
3 × 1·5 cm	4 × 1·5 cm	5 × 1·5 cm

INVESTIGATION 3:01A RIGHT-ANGLED TRIANGLES

1 Take three lengths of rope or string 3 m, 4 m and 5 m in length. Use them to form a triangle, and measure the largest angle. Is it a right angle? (You could use a piece of string 12 m long and use marks to show lengths of 3 m, 4 m, and 5 m.)



3:01 Content statements

Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles (**ACMMG222**)

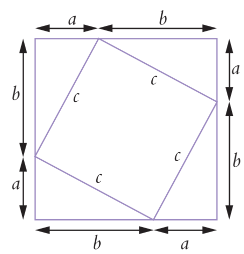
- identify the hypotenuse as the longest side in any right-angled triangle and also as the side opposite the right angle
- establish the relationship between the lengths of the sides of a right-angled triangle in practical ways, including with the use of digital technologies

Lesson starter

My squares

Pair students for this activity. Construct and label two squares with areas of 25 cm^2 and 49 cm^2. Have the students investigate how they could fit the smaller square into the larger square so that all four corners of the small square touch the edges of the larger square. Get students to measure the sides of the triangles created within the larger square. If they have constructed the squares correctly, the students will find that the corners of the smaller square touch the larger square exactly 3 cm and 4 cm from the same corner of the large square. Ask students to try another two squares of 100 cm^2 and 196 cm^2.

Answer:



Class activities

Investigation 3:01A

It is important for students to investigate right-angled triangles based on lengths of 3, 4 and 5 units. Ideally, have various right-angled triangles with side lengths that are Pythagorean triads. This will enable students to measure the side lengths and discover the relationship for themselves.

Complete Investigation 3:01A as a class. Provide time during and after the activity for discussion and evaluation.

[Understanding, Fluency]

P Digital resources

eBook

- Foundation worksheet 3:01 Pythagoras' theorem 1
- Perigal's dissection (GeoGebra)

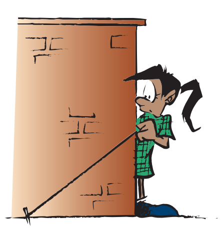
Technology

Interactive Pythagoras' theorem

Students will benefit from a visual investigation of Pythagoras' theorem to gain a greater understanding that the square on the hypotenuse is equal to the sum of the squares on the smaller sides.

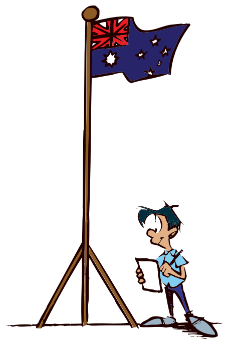
Search the internet for 'Interactive Pythagoras' theorem'. Students can see how the theorem works by viewing and exploring with various interactive activities.

2 Measure 1·5 m up and 2 m along the bottom of a wall that stands on flat ground. How long is the distance between these two points? Use string to measure this distance. Is the triangle a {3, 4, 5} triangle?



3 Draw or find some right-angled triangles in the school grounds. Make the sides any length you wish. For each triangle, measure the side lengths a, b and c (where c is the longest side) and enter these in a table like the one below. Make sure you include the three sets of measurements in the table.

a	3	6	5			
b	4	8	12			
c						

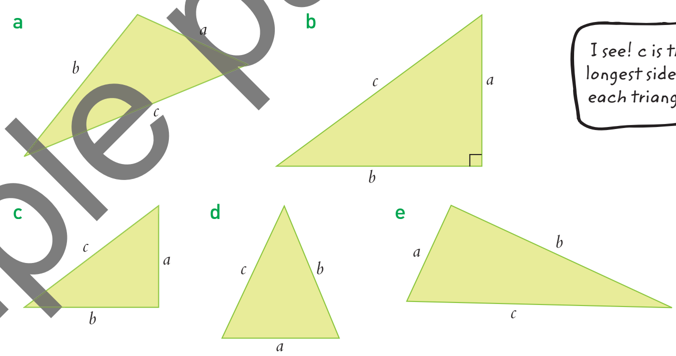


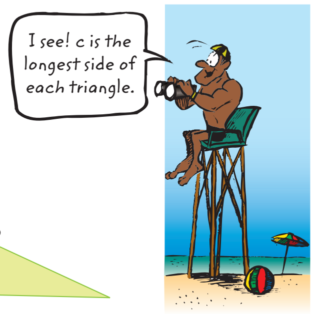
Exercise 3:01

Foundation worksheet 3:01 Pythagoras' theorem A

1 For each of the triangles drawn below, measure the sides to the nearest millimetre and complete the table.

a	b	c	a^2	b^2	c^2	$a^2 + b^2$





2 In which of the triangles in Question **1** does $c^2 = a^2 + b^2$? What type of triangles are they?

Answers

Exercise 3:01

		a	b	c	a^2	b^2	c^2	$a^2 + b^2$
1	a	2	3	4	4	9	16	13
	b	3	4	5	9	16	25	25
	c	2·4	3·2	4	5·76	10·24	16	16
	d	2·5	3	3	6·25	9	9	15·25
	e	2	4·8	5·2	4	23·04	27·04	27·04

2 **b**, **c**, **e**, right-angled triangles

Teaching strategies

Area of square

Students may need to be reminded that the formula for finding the area of a square is $A = l^2$.

a^2 and b^2

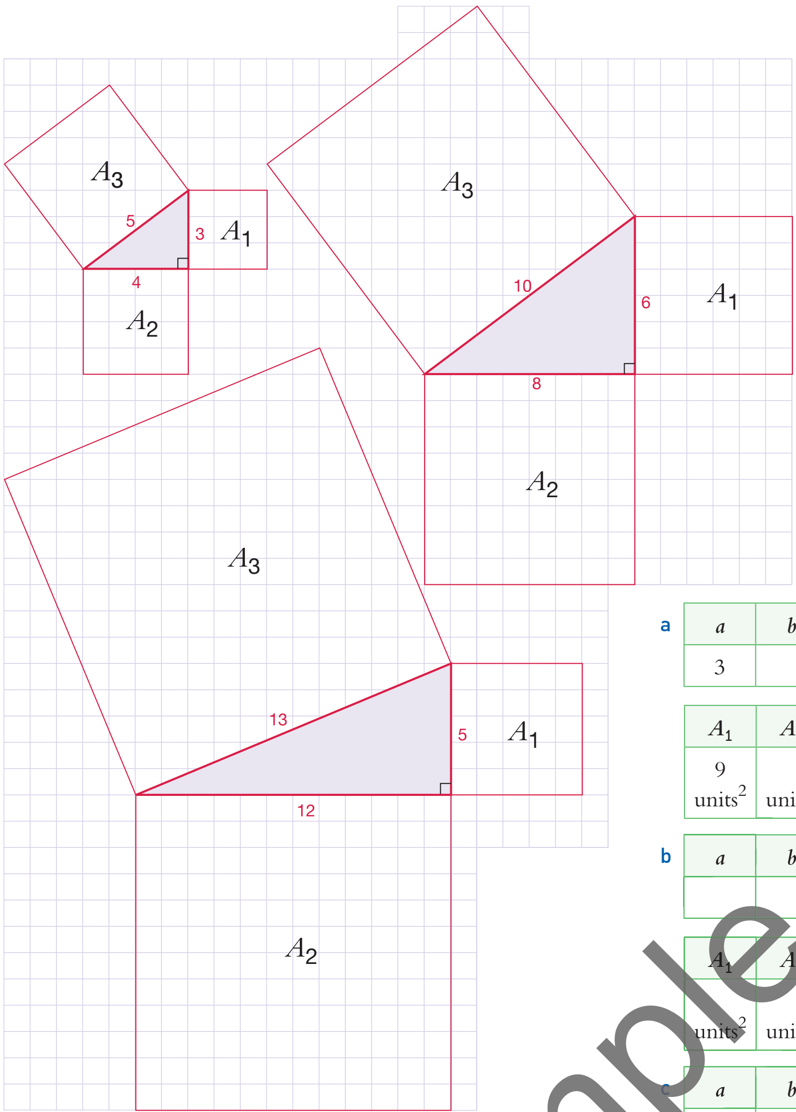
Remind students that a^2 is $a \times a$.
For example: $3^2 = 3 \times 3$.

Relationship between *a*, *b*, and *c*

Have students complete Questions 1 and 2 to find and explain the relationship between a, b and c in right-angled triangles.

Teacher's notes

3 A square has been drawn on each of the sides of these right-angled triangles. Complete the tables for each triangle. Can you find a rule that links *a*, *b* and *c*?



a

a	*b*	*c*
3		

A_1	A_2	A_3
9 units²	units²	units²

b

a	*b*	*c*

A_1	A_2	A_3
units²	units²	units²

c

a	*b*	*c*

A_1	A_2	A_3
units²	units²	units²

Answers

Exercise 3:01

3 a

a	*b*	*c*
3	4	5

A_1	A_2	A_3
9	16	25

b

a	*b*	*c*
6	8	10

A_1	A_2	A_3
36	64	100

c

a	*b*	*c*
5	12	13

A_1	A_2	A_3
25	144	169

Class activities

What is my rule?

Playing this game provides an opportunity for students to see patterns when adding squares of numbers.

Step 1: Give students a copy of these numbers:

2 5 8 10 13 17 18 20 25 26 29 34 37 40 41 45 50 52 53 58 61 65 68 72 73 74 80 82 85 86 89 90 97 98 100 106 113 117 128 130 145 162

Step 2: Ask a student to choose one of the numbers.

Step 3: The square of which two numbers is equal to the chosen number? (e.g. The rule for 80 is $8^2 + 4^2 = 80$)

Step 4: Students find the two numbers and raise their hand to state the rule. The first student with the correct answer chooses the next number from the list.

[Understanding, Fluency]

Answer:

Number		1	2	3	4	5	6	7	8	9
	Squares	1	4	9	16	25	36	49	64	81
1	1	2	5	10	17	26	37	50	65	82
2	4		8	13	20	29	40	53	68	86
3	9			18	25	34	45	58	73	90
4	16				32	41	52	65	80	97
5	25					50	61	74	89	106
6	36						72	85	100	117
7	49							98	113	130
8	64								128	145
9	81									162

Answers

Exercise 3:01

4 a

Triangle	AB^2	(right side)^2	(left side)^2
ΔABC	16	9	$(2{\cdot}5)^2 = 6{\cdot}25$
ΔABD	16	9	16
ΔABE	16	9	25
ΔABF	16	9	36

b ΔABE c 90°

Teaching strategies

Investigation 3:01B

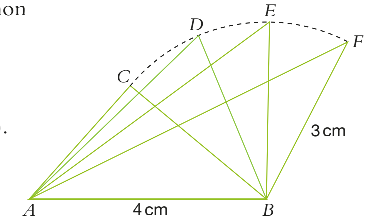
This investigation is designed to help students establish Pythagoras' theorem. Guide the students to use values of $a = 3$ and $b = 4$, or $a = 6$ and $b = 8$, or $a = 9$ and $b = 12$ (Pythagorean triads) as this will assist them to work out the value of c.

Answers

INVESTIGATION 3:01B

a yes
b yes
c yes
d $a^2 + b^2$
e c^2
f yes
In any right-angled triangle the square on the longest side is equal to the sum of the squares on the other two sides.

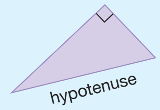
4 A sequence of triangles has been formed with a common base AB by gradually increasing the size of $\angle B$ while keeping the side BC constant in length. The vertex C moves along a circle to the positions D, E and F. This concept is demonstrated in GeoGebra activity 3:03 (1).



a Complete the table for the triangles named.

Triangle	AB^2	(right side)^2	(left side)^2
ΔABC			
ΔABD			
ΔABE			
ΔABF			

The longest side of a right-angled triangle is called the hypotenuse.

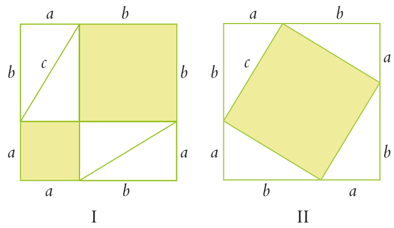


b For which triangle does AB^2 + (right side)^2 = (left side)^2?
c What is the size of $\angle B$ for the triangle in **b**?

While answering the questions above you should have discovered, like the Pythagoreans, the relationship between the sides of a right-angled triangle.

INVESTIGATION 3:01B PYTHAGORAS' THEOREM

Construct two identical squares as shown in the diagram. Use any value you like for a and b (e.g. $a = 6$ cm, $b = 8$ cm). The coloured squares are built on the three sides of a right-angled triangle that has side lengths a, b and c.



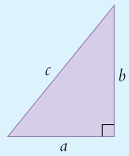
a Is the area of square I equal to the area of square II?
b Do the triangles in square I take up the same areas as the triangles in square II?
c If the triangles are removed from squares I and II, are the remaining parts equal in area?
d If the triangles are removed from square I, what is the area of the two remaining squares?
e If the triangles are removed from square II, what is the area of the remaining square?
f Does $c^2 = a^2 + b^2$?
Complete: In any right-angled triangle the square on the ____________ side is equal to the _________ of the squares on the other two _________ .

Teacher's notes

Pythagoras' theorem

- In any right-angled triangle the square of the longest side is equal to the sum of the squares of the two shorter sides. For the triangle shown, the theorem can be written as:

  $c^2 = a^2 + b^2$



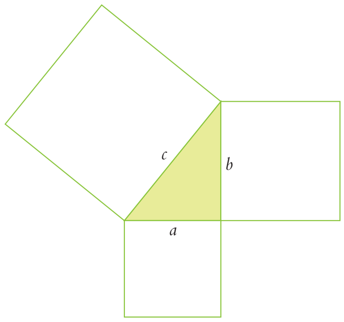
Converse

- If a triangle obeys the equation above, it must be right-angled.

Class activity

Pythagoras' theorem can also be illustrated by the following geometric method.

Construct a right-angled triangle and construct squares on each side of the triangle as shown in the diagram.



We have seen that for all right-angled triangles:

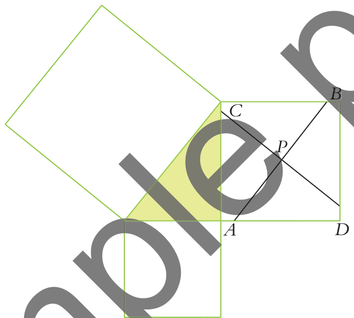
$c^2 = a^2 + b^2$

The areas of the squares constructed on the sides are a^2, b^2 and c^2 respectively, so the theorem can be restated as:

The square on the hypotenuse is equal to the sum of the squares on the two smaller sides.

The larger square can be made from the two smaller squares by dissecting the middle square and rearranging these pieces with the small square. The dissection is shown in the following diagram. Copy the dissection and find how the pieces can be arranged to form the large square.

- *P* is the centre of the square.
- *AB* is drawn through *P*, parallel to the hypotenuse.
- *CD* is drawn through *P*, perpendicular to the hypotenuse.



P GEOGEBRA ACTIVITY 3:01 PERIGAL'S DISSECTION

Complete this GeoGebra activity on Perigal's dissection to further your investigations on Pythagoras' theorem.

Class activities

Triangle dissection

Students will need: scissors, paper, ruler, glue

Ask students to complete the activity in the Student Book. On paper, they draw a right-angled triangle and then draw squares on each side of the triangle. The two smaller squares are then dissected to make puzzle pieces that fit together to create the largest square. Have students glue the resulting puzzle into their workbooks.

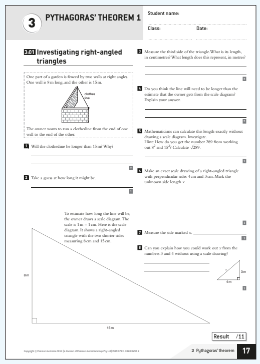
Discuss as a class what they have learnt from this activity. Have students write a concluding paragraph in their workbooks about how this activity relates to Pythagoras' theorem.

[Understanding, Reasoning]

Interesting fact

There are hundreds of known proofs for Pythagoras' theorem. This is more than for any other theorem in mathematics.

Homework 3:01



Teacher's notes

Technology

GeoGebra activity 3:01

Use the GeoGebra activity to develop the students' knowledge on how Pythagoras' theorem works using Perigal's dissection. Perigal cut the larger square into regions that could be reassembled to make the smaller squares.

3:02 Content statements

Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles (**ACMMG222**)

- identify the hypotenuse as the longest side in any right-angled triangle and also as the side opposite the right angle
- use Pythagoras' theorem to find the length of an unknown side in a right-angled triangle
- solve a variety of practical problems involving Pythagoras' theorem, approximating the answer as a decimal

Investigate the concept of irrational numbers (**ACMNA186**)

- solve a variety of practical problems involving Pythagoras' theorem, giving exact answers (i.e. as surds where appropriate), eg $\sqrt{5}$

Lesson starter

Finding the hypotenuse

Draw a range of different right-angled triangles on the board or reproduce them on a whiteboard.

Have students label the triangles with the pronumerals *a*, *b*, and *c* (hypotenuse). This will give them practice before using Pythagoras' theorem.

Teaching strategies

Label legs and hypotenuse

When calculating the length of the hypotenuse, ask students to:

- redraw the triangle
- label the two legs from the right angle as *A* and *B*
- draw an arrow from 90° to the hypotenuse.

Answers

PREP QUIZ 3:02

1 A	2 B	3 B	4 $c^2 = n^2 + m^2$
5 4	6 25	7 25	8 25
9 6	10 no		

Digital resources

eBook

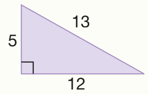
- Foundation worksheet 3:02 Pythagoras' theorem 2

3:02 Pythagoras' theorem: Calculating the hypotenuse

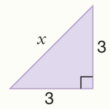
PREP QUIZ 3:02

For the triangles given in **1** to **3**, select the correct statements from **A**, **B** or **C**.

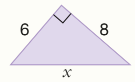
1 A $5^2 + 12^2 = 13^2$
B $(5 + 12)^2 = 13^2$
C $13^2 + 5^2 = 12^2$



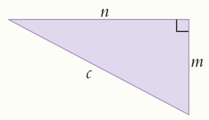
2 A $3 + 3 = x$
B $3^2 + 3^2 = x^2$
C $(3 + 3)^2 = x^2$



3 A $8^2 = 6^2 + x^2$
B $x^2 = 6^2 + 8^2$
C $x = 6 + 8$



4 Write Pythagoras' theorem for the triangle given.



The side length of a triangle must be positive.

5 If $4^2 = 16$, then $\sqrt{16}$ = ______

6 If $25^2 = 625$, then $\sqrt{625}$ = ______

7 If $\sqrt{25} = 5$, then 5^2 = ______

8 $3^2 + 4^2$ = ______

9 $\sqrt{36}$ = ______

10 Can 2, 3 and 4 be the side lengths of a right-angled triangle?

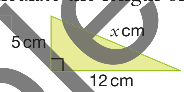
Pythagoras' theorem is used to calculate the lengths of the sides of right-angled triangles. To calculate one side, the other two sides must be known.

- $x^2 = 81$ has two solutions, $x = 9$ and $x = -9$. As side lengths must be positive we ignore the negative solution when finding a length.
- If $x^2 = c$ (where $x > 0$ and $c > 0$), then $x = \sqrt{n}$.

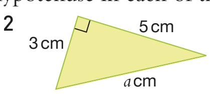
WORKED EXAMPLES

Calculate the length of the hypotenuse in each of the following triangles.

1



2

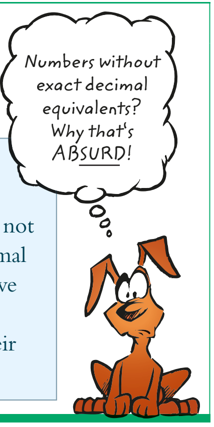


Solutions

1 $x^2 = 5^2 + 12^2$
$= 25 + 144$
$= 169$
$x = \sqrt{169}$
$= 13$
Hypotenuse is 13 cm.

2 $a^2 = 3^2 + 5^2$
$= 9 + 25$
$= 34$
$a = \sqrt{34}$
$a \doteqdot 5{\cdot}8$ (1 dec. pl.)
Hypotenuse is $\sqrt{34}$ cm.

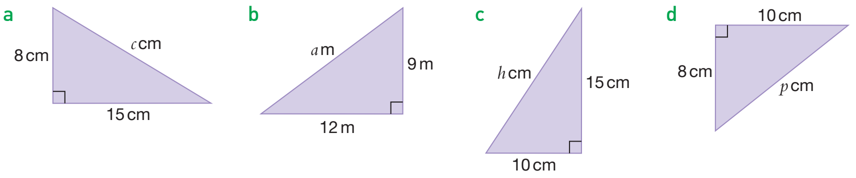
Numbers like $\sqrt{34}$ are called surds. **Surds** do not have exact decimal equivalents, so we use decimals to approximate their value.



Teacher's notes

Exercise 3:02A (Calculators are not needed)

1 Use Pythagoras' theorem to write an equation that shows how the sides of each triangle are related to each other.



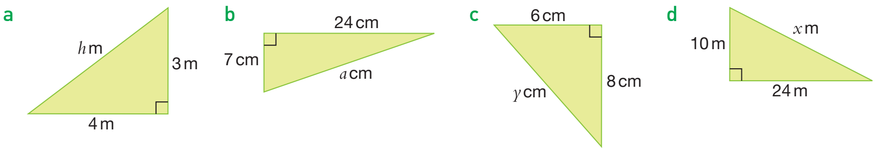
2 Complete the following table.

Number	3	4	5	6	7	8	9	10	12	13	15	16	17	18	20	24	25	26	30
Square	9	16	25																

3 Use the table in Question 2 to find:

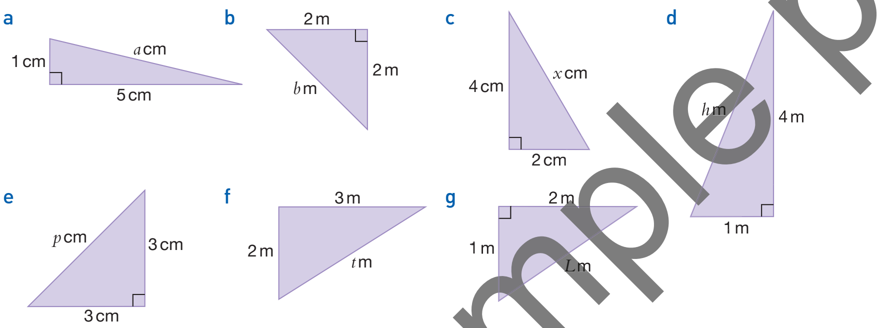
a $\sqrt{25}$ b $\sqrt{100}$ c $\sqrt{169}$ d $\sqrt{225}$ e $\sqrt{900}$ f $\sqrt{625}$ g $\sqrt{289}$

4 Find the length of the hypotenuse in each of the following. (Use the table in Question 2.)



5 Find the length of the hypotenuse in each of the following. (Use the table below to evaluate the surds, correct to one decimal place.)

Square root	$\sqrt{5}$	$\sqrt{8}$	$\sqrt{10}$	$\sqrt{13}$	$\sqrt{17}$	$\sqrt{18}$	$\sqrt{20}$	$\sqrt{26}$	$\sqrt{29}$	$\sqrt{32}$	$\sqrt{34}$
Approximate value	2·2	2·8	3·2	3·6	4·1	4·2	4·5	5·1	5·4	5·7	5·8



Teacher's notes

Teaching strategies

Substitution and Pythagoras' theorem

Students must practise substituting values into $c^2 = a^2 + b^2$ to gain confidence when using the rule.

Surds and Pythagoras' theorem

Students must practise using a calculator when dealing with surds and decimal places. Teachers may need to revise rounding decimal places for some students.

Answers

Exercise 3:02A

1 a $c^2 = 8^2 + 15^2$ b $a^2 = 9^2 + 12^2$
c $h^2 = 10^2 + 15^2$ d $p^2 = 8^2 + 10^2$

2

Number	3	4	5	6	7	8	9	10	12
Square	9	16	25	36	49	64	81	100	144
Number	13	15	16	17	18	20	24	25	26
Square	169	225	256	289	324	400	576	625	676

3 a 5 b 10 c 13 d 15
e 30 f 25 g 17

4 a 5 m b 25 cm c 10 cm d 26 m

5 a 5·1 cm b 2·8 m c 4·5 cm d 4·1 m
e 4·2 cm f 3·6 m g 2·2 m