ALGEBRA

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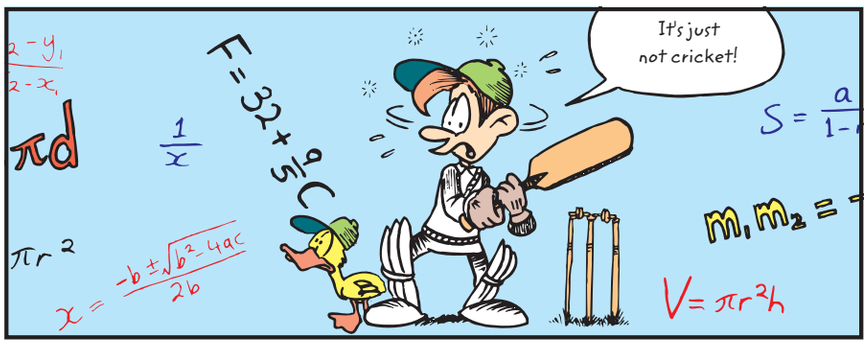


Contents

Syllabus references (See pages x–xv for details.)

Number and Algebra

Selections from ***Algebraic Techniques*** [Stages 5.2, 5.3§]

- Simplify algebraic expressions involving the four operations [Stage 4]
- Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (ACMNA213)
- Apply the four operations to simple algebraic fractions with numerical denominators (ACMNA232)
- Factorise algebraic expressions by taking out a common algebraic factor (ACMNA230)
- Add and subtract algebraic fractions with numerical denominators, including those with binomial numerators (NSW)
- Expand binomial products using a variety of strategies (ACMNA233)

Working Mathematically

• Problem Solving • Reasoning • Understanding • Fluency • Communicating

CHAPTER FOCUS

This chapter builds on Stage 4 algebra concepts, starting with simplification of algebraic expressions involving the four operations. Integers and rational numbers are then presented. Students simplify algebraic fractions by cancelling common factors. Expanding expressions using the distributive law leads to factorising, again using common factors.

Outcomes

Algebraic Techniques [Stages 5.2, 5.3§]

MA4-1WM communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols

MA4-3WM recognises and explains mathematical relationships using reasoning

MA4-8NA generalises number properties to operate with algebraic expressions

MA5.2-1WM selects appropriate notations and conventions to communicate mathematical ideas and solutions

MA5.2-3WM constructs arguments to prove and justify results

MA5.2-6NA simplifies algebraic fractions, and expands and factorises quadratic expressions

MA5.3-1WM uses and interprets formal definitions and generalisations when explaining solutions and/or conjectures

MA5.3-5NA selects and applies appropriate algebraic techniques to operate with algebraic expressions

Key ideas

- In algebra, only like terms can be added or subtracted. Like terms contain identical pronumeral parts.
- To multiply algebraic terms, multiply numbers first and then pronumerals.
- To divide algebraic terms, express the number as a fraction and if possible, cancel the numbers and pronumerals.
- Always look for the lowest common denominator when adding or subtracting algebraic fractions.
- Use the distributive law to expand an expression and, to do this in reverse, factorise the expanded expression.
- To factorise, always look for the highest common factor.

Language

algebraic expression
algebraic fraction
common factor
denominator
distributive law
expand
factorisation
grouping symbols
highest common factor
like terms
numerator
parentheses
pronumeral
simplest form
simplify

3:01 Content statements

The following Stage 4 content is addressed in this exercise.

Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)

Simplify algebraic expressions involving the four operations (ACMNA192)

Answers

PREP QUIZ 3:01

1 $9x$	2 x	3 $6xy$	4 $5x^2$
5 $3x$	6 $2b$	7 $8a + 5b$	8 $5x + y$
9 $-24a^2$	10 $-\frac{a}{3b}$		

Lesson starter

Prep quiz 3:01

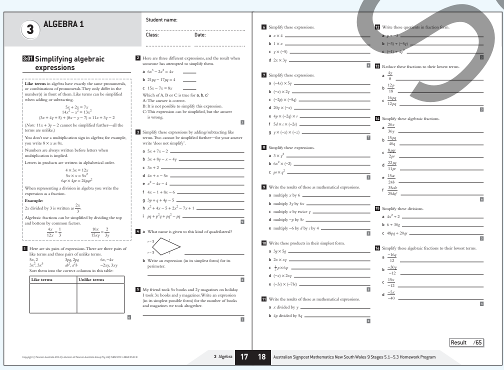
Use this as a lesson starter. Basic questions from Exercise 3:01 (1 a–h) could also be included and presented as a starter quiz.

Teaching strategies

My like and unlike terms

To build confidence with adding and subtracting pronumerals, have students sort the expression into a table with headings 'Like terms' and 'Unlike terms'. This encourages students to group the pronumerals before adding or subtracting. Remind students that sometimes expressions cannot be simplified any further.
For example: $3x + 2y - 6a$.

Homework 3:01



3:01 Simplifying algebraic expressions

PREP QUIZ 3:01

Simplify the following.

1 $7x + 2x$	2 $9x - 8x$	3 $3x \times 2y$	4 $5x \times x$
5 $12x \div 4$	6 $10ab \div 5a$	7 $3a + 2b + 5a + 3b$	8 $6x + 2y - x - y$
9 $3 \times (-2a) \times 4a$	10 $3a \div (-9b)$		

WORKED EXAMPLES

Remember that only like terms may be added or subtracted.

1 $5a + 2b - 3a + b = 5a - 3a + 2b + b$
$= 2a + 3b$

2 $5p^2 + 2p - 3p^2 = 5p^2 - 3p^2 + 2p$ (p^2 and p are not like terms.)
$= 2p^2 + 2p$

3 $6ab - 4ba = 6ab - 4ab$
$= 2ab$

4 $-7x \times -3xy^2 = 21x^2y^2$

5 $3pq \times 4qr = 12pqqr$
$= 12pq^2r$

6 $12ac \div 8ab = \frac{^{3}\cancel{12}\,\cancel{a}c}{_{2}\cancel{8}\,\cancel{a}b}$
$= \frac{3c}{2b}$

7 $-6x \div 18xy = \frac{-^{1}\cancel{6}\,\cancel{x}}{_{3}\cancel{18}\,\cancel{x}y}$
$= -\frac{1}{3y}$

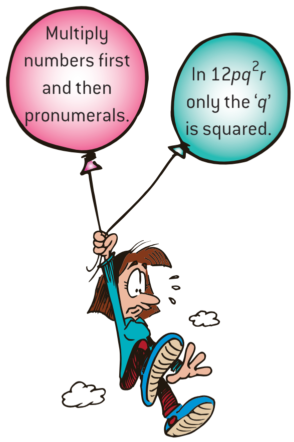
8 $10a - 3 \times 2a = 10a - 6a$
$= 4a$

9 $(5a + 7a) \times (3b - 2b) = 12a \times b$
$= 12ab$

10 $3m \times 2n \div mn = 6mn \div mn$
$= \frac{6\,\cancel{mn}^{1}}{\cancel{mn}_{1}}$
$= 6$

11 $\frac{7p + 8p - 3p}{2p \times 3q} = \frac{^{2}\cancel{12}\,\cancel{p}}{_{1}\cancel{6}\,\cancel{p}q}$
$= \frac{2}{q}$

Remember the order in which operations should be done.
↓
Grouping symbols
↓
× ÷
↓
+ −

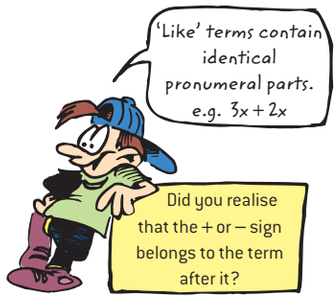


Teacher's notes

Exercise 3:01

1 Collect the like terms to simplify these expressions.

a $3x + 2x$	**b** $8a + 5a$		
c $10p + 21p$	**d** $x + 7x$		
e $7a - 4a$	**f** $9b - 3b$		
g $11q - q$	**h** $12e + 9e$		
i $3p + 5p - 6p$	**j** $4x + 2x + x$		
k $10x - 9x + 3x$	**l** $x + 2x - 3x$		
m $2a + p - a + 3p$	**n** $a + m - a + m$		
o $8 + 2x - 5x - 7$	**p** $8y - 1 - 8y - 1$		
q $x^2 + 2x + 2x^2 - x$	**r** $p^2 + 4p + 3p^2 + p$		
s $3q^2 + 8q - 4q - q^2$	**t** $y^2 + y + y^2 - y$		
u $7 - p^2 + p - 5$	**v** $2a + a^2 + 7 + a$		
w $8x - 7 - 7x - 3x^2$	**x** $5ab - 7 + 3ba - 9$		



2 Simplify these products.

a $8y \times 3$	**b** $4 \times 4a$	**c** $3x \times 2y$	**d** $8p \times 4q$
e $6a \times b$	**f** $5x \times x$	**g** $5a \times 3a$	**h** $ab \times ac$
i $3pq \times 2p$	**j** $5mn \times mp$	**k** $4mn \times \frac{1}{2}n$	**l** $9b \times a^2$
m $6a^2 \times (-7a)$	**n** $-5x \times -2x$	**o** $x \times 2y \times 3x$	**p** $14ab \times (-\frac{1}{2}ab)$
q $(-ab) \times (-bc)$	**r** $2k \times 3k \times 4k$	**s** $-2 \times 7x \times -5y$	**t** $\frac{1}{4}m \times 4n \times (-p)$

3 Simplify:

a $12x \div 4$	**b** $12x \div 4x$	**c** $9x^2 \div 3$	**d** $8x \div 8x$
e $15m \div 10n$	**f** $32a \div 12b$	**g** $5 \div 20a$	**h** $48ab \div 6b$
i $a \div 3a$	**j** $45ab \div 20ba$	**k** $-20p \div 4p$	**l** $-xy \div xz$
m $14a \div (-a)$	**n** $(-15x) \div (-5xy)$	**o** $-28mnp \div 7mp$	**p** $8a^2b \div 16ab^2$

4 Simplify:

a $mn \times np$	**b** $7 + m + 6 + 3m$	**c** $14 - 2a + 5$	**d** $5x^2 \times 0$
e $3xy \times 2yx$	**f** $8x^2 + 2x + 7x^2 + 3x$	**g** $3 \times 4y \times 5z$	**h** $-4x \times 7x$
i $15ab - 9ba + ab$	**j** $6m - 7m$	**k** $8b + 3b - 11b$	**l** $18ab \div 9bc$
m $x \div 3x$	**n** $2pq \times 9pq$	**o** $3a + b + 2a - c$	**p** $-3y \times (-5z)$
q $\frac{1}{2}y + \frac{1}{2}y$	**r** $m + n - m + n$	**s** $3a \times 2b \times c$	**t** $15at \div 10tx$

5 Write the simplest expression for:

a $(2a + 3a) \times 4$	**b** $(10x - 3x) \div 7$	**c** $(9b - 3b) \times 2$	**d** $(3m + 9m) \div 4$
e $12x \div (2x + x)$	**f** $5a \times (10a + 2a)$	**g** $3m \times (10m - 9m)$	**h** $15y \div (9y - 2y)$
i $5a \times 7 \div a$	**j** $8x \times 4y \div 2xy$	**k** $10a \div 5 \times 3a$	**l** $9xy \div 3x \times 2y$
m $2x + 3x \times 4$	**n** $5x \times 3x + 10x^2$	**o** $20y - 5 \times 2y$	**p** $18m - 12m \div 6$
q $3 \times 2n + 5n \times 4$	**r** $7x + 3 \times 2x - 10x$	**s** $8x \div 4 - x$	**t** $11m + 18m \div 2$
u $\frac{6 \times 3x}{2x \times 5}$	**v** $\frac{3p + 2p - 1p}{2 \times 2p}$	**w** $\frac{11y - y}{6y + 4y}$	**x** $\frac{5a \times 4b \times 2c}{10c \times b \times 8c}$

Teaching strategies

Multiplication in algebra

Emphasise that when multiplying pronumerals the terms do not have to be the same. Have students follow these steps.

Step 1 Write the terms.

Step 2 Rearrange the expression so that the numbers are at the start and pronumerals are at the end.

Step 3 Calculate the numbers and then write the pronumerals in alphabetical order without the multiplication sign.

Multiplication symbol

In algebra, we don't write the multiplication symbol. For example, $13 \times w \times y$ is written as $13wy$.

Division in algebra

Step 1 Write the division as a fraction.

Step 2 Try to find the highest common factor and then cancel.

Step 3 Cancel any pronumerals in the numerator and denominator.

Step 4 Check that you have fully simplified the expression.

Answers

Exercise 3:01

1 a $5x$	**b** $13a$		
c $31p$	**d** $8x$		
e $3a$	**f** $6b$		
g $10q$	**h** $21e$		
i $2p$	**j** $7x$		
k $4x$	**l** 0		
m $a + 4p$	**n** $2m$		
o $1 - 3x$	**p** -2		
q $3x^2 + x$	**r** $4p^2 + 5p$		
s $2q^2 + 4q$	**t** $2y^2$		
u $2 - p^2 + p$	**v** $3a + a^2 + 7$		
w $x - 7 - 3x^2$	**x** $8ab - 16$		
2 a $24y$	**b** $16a$		
c $6xy$	**d** $32pq$		
e $6ab$	**f** $5x^2$		
g $15a^2$	**h** a^2bc		
i $6p^2q$	**j** $5m^2np$		
k $2mn^2$	**l** $9a^2b$		
m $-42a^3$	**n** $10x^2$		
o $6x^2y$	**p** $-7a^2b^2$		
q ab^2c	**r** $24k^3$		
s $70xy$	**t** $-mnp$		
3 a $3x$	**b** 3		
c $3x^2$	**d** 1		
e $\frac{3m}{2n}$	**f** $\frac{8a}{3b}$		
g $\frac{1}{4a}$	**h** $8a$		
i $\frac{1}{3}$	**j** $\frac{9}{4}$		
k -5	**l** $-\frac{y}{z}$		
m -14	**n** $\frac{3}{y}$		
o $-4n$	**p** $\frac{a}{2b}$		
4 a mn^2p	**b** $4m + 13$		
c $19 - 2a$	**d** 0		
e $6x^2y^2$	**f** $15x^2 + 5x$		
g $60yz$	**h** $-28x^2$		
i $7ab$	**j** $-m$		
k 0	**l** $\frac{2a}{c}$		
m $\frac{1}{3}$	**n** $18p^2q^2$		
o $5a + b - c$	**p** $15yz$		
q y	**r** $2n$		
s $6abc$	**t** $\frac{3a}{2x}$		
5 a $20a$	**b** x		
c $12b$	**d** $3m$		
e 4	**f** $60a^2$		
g $3m^2$	**h** $\frac{15}{7}$		
i 35	**j** 16		
k $6a^2$	**l** $6y^2$		
m $14x$	**n** $25x^2$		
o $10y$	**p** $16m$		
q $26n$	**r** $3x$		
s x	**t** $20m$		
u $\frac{9}{5}$	**v** 1		
w 1	**x** $\frac{a}{2c}$		

3:02 Content statements

Apply the four operations to simple algebraic fractions with numerical denominators (ACMNA232) [Stage 5.2]

- simplify expressions that involve algebraic fractions with numerical denominators, eg $\frac{a}{2}+\frac{a}{3}, \frac{2x}{5}-\frac{x}{3}, \frac{3x}{4}\times\frac{2x}{9}, \frac{3x}{4}\div\frac{9x}{2}$

Lesson starter

Adding and subtracting fractions

Revise fractions with the following quick quiz presented on the board.

1 $\frac{1}{3}+\frac{1}{3}$ **2** $\frac{3}{5}-\frac{2}{5}$
3 $\frac{7}{10}+\frac{1}{10}$ **4** $\frac{9}{10}-\frac{3}{10}$
5 $\frac{1}{3}+\frac{1}{2}$ **6** $\frac{1}{2}-\frac{1}{3}$
7 $\frac{2}{3}+\frac{1}{6}$ **8** $\frac{1}{6}+\frac{1}{9}$
9 $\frac{5}{6}-\frac{1}{4}$ **10** $\frac{7}{12}-\frac{3}{8}$

Answers:

1 $\frac{2}{3}$ **2** $\frac{1}{5}$
3 $\frac{8}{10}=\frac{4}{5}$ **4** $\frac{6}{10}=\frac{3}{5}$
5 $\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$ **6** $\frac{3}{6}-\frac{2}{6}=\frac{1}{6}$
7 $\frac{4}{6}+\frac{1}{6}=\frac{5}{6}$ **8** $\frac{3}{18}+\frac{2}{18}=\frac{5}{18}$
9 $\frac{10}{12}-\frac{3}{12}=\frac{7}{12}$ **10** $\frac{14}{24}-\frac{9}{24}=\frac{5}{24}$

Answers

PREP QUIZ 3:02A

1 $\frac{4}{5}$ **2** $\frac{2}{5}$ **3** $\frac{7}{12}$ **4** $\frac{7}{8}$
5 $\frac{3}{20}$ **6** $\frac{1}{4}$ **7** $11x$ **8** $4ab$
9 x **10** $8a$

Digital resources

eBook
- Foundation worksheet 3:02A Simplifying algebraic fractions
- Foundation worksheet 3:02B Simplifying algebraic fractions

ProductLink
- Addition and subtraction of algebraic fractions (Drag-and-drop)
- Multiplication and division of algebraic fractions (Drag-and-drop)

3:02 Algebraic fractions

3:02A Addition and subtraction

PREP QUIZ 3:02A

Answer the following:

1 $\frac{3}{5}+\frac{1}{5}$ **2** $\frac{7}{10}-\frac{3}{10}$ **3** $\frac{1}{4}+\frac{1}{3}$
4 $\frac{1}{2}+\frac{3}{8}$ **5** $\frac{2}{5}-\frac{1}{4}$ **6** $\frac{7}{12}-\frac{1}{3}$

Simplify the expressions:

7 $7x+4x$ **8** $3ab+ab$ **9** $6x-5x$ **10** $9a-a$

Rewrite each fraction as two equivalent fractions with a common denominator, then add or subtract the numerators.

WORKED EXAMPLES

1 $\frac{3x}{5}+\frac{2x}{5}=\frac{3x+2x}{5}$
$=\frac{{}^{1}\cancel{5}x}{{}_{1}\cancel{5}}$
$=x$

2 $\frac{5}{a}-\frac{3}{a}=\frac{5-3}{a}$
$=\frac{2}{a}$

3 $\frac{x}{3}+\frac{x}{2}=\frac{x\times2}{3\times2}+\frac{x\times3}{2\times3}$
$=\frac{2x}{6}+\frac{3x}{6}$
$=\frac{5x}{6}$

4 $\frac{4a}{5}-\frac{a}{3}=\frac{4a\times3}{5\times3}-\frac{a\times5}{3\times5}$
$=\frac{12a}{15}-\frac{5a}{15}$
$=\frac{7a}{15}$

5 $\frac{5m}{8}+\frac{m}{2}=\frac{5m}{8}+\frac{m\times4}{2\times4}$
$=\frac{5m}{8}+\frac{4m}{8}$
$=\frac{9m}{8}$

6 $\frac{3x}{4}-\frac{2y}{3}=\frac{9x}{12}-\frac{8y}{12}$
$=\frac{9x-8y}{12}$

7 $\frac{9}{x}+\frac{2}{3x}=\frac{27}{3x}+\frac{2}{3x}$
$=\frac{29}{3x}$

8 $\frac{5a}{2x}-\frac{2a}{3x}=\frac{15a}{6x}-\frac{4a}{6x}$
$=\frac{11a}{6x}$

Teacher's notes

Exercise 3:02A

P Foundation worksheet 3:02A
Simplifying algebraic fractions

1 Simplify the following.

a $\frac{3a}{2}+\frac{a}{2}$ b $\frac{3x}{5}-\frac{2x}{5}$ c $\frac{a}{3}+\frac{4a}{3}$ d $\frac{9m}{10}-\frac{3m}{10}$

e $\frac{x}{4}+\frac{y}{4}$ f $\frac{5a}{3}-\frac{2b}{3}$ g $\frac{2}{a}+\frac{3}{a}$ h $\frac{7}{x}+\frac{1}{x}$

i $\frac{3}{y}-\frac{2}{y}$ j $\frac{9}{m}-\frac{1}{m}$ k $\frac{5a}{x}+\frac{2a}{x}$ l $\frac{2x}{y}-\frac{3x}{y}$

m $\frac{5}{3n}+\frac{7}{3n}$ n $\frac{3}{2x}-\frac{1}{2x}$ o $\frac{8a}{5b}+\frac{2a}{5b}$ p $\frac{7m}{4x}-\frac{3m}{4x}$

2 Reduce each of these expressions to its simplest form.

a $\frac{x}{3}+\frac{x}{5}$ b $\frac{a}{2}+\frac{a}{5}$ c $\frac{y}{3}-\frac{y}{4}$ d $\frac{m}{2}-\frac{m}{4}$

e $\frac{2a}{3}+\frac{a}{2}$ f $\frac{5x}{3}+\frac{2x}{4}$ g $\frac{3n}{8}-\frac{n}{4}$ h $\frac{4p}{5}-\frac{3p}{10}$

i $\frac{x}{4}+\frac{y}{3}$ j $\frac{2a}{3}-\frac{3b}{2}$ k $\frac{3m}{5}+\frac{n}{2}$ l $\frac{k}{6}-\frac{21}{4}$

m $\frac{2}{x}+\frac{4}{3x}$ n $\frac{1}{3a}+\frac{2}{4a}$ o $\frac{7}{2m}-\frac{2}{5m}$ p $\frac{5}{8x}-\frac{1}{2x}$

q $\frac{2a}{3x}+\frac{3a}{2x}$ r $\frac{x}{3m}-\frac{2x}{m}$ s $\frac{5m}{2n}+\frac{3m}{4n}$ t $\frac{2x}{3a}+\frac{y}{4a}$

FUN SPOT 3:02 — TRY THIS MATHS-WORD PUZZLE

Hidden in the maze of letters there are many words used in mathematics. Make a list of the words you find and, at the same time, put a line through the letters you use. Words may be written in any direction: up, down, backwards, even diagonally. Also, a letter may be used more than once, but you cannot change direction in order to form a word (i.e. the letters must be in a straight line).

When you have found all the words there should be four letters that have not been used. These four letters can be arranged to form another 'mystery' maths word.

R	E	T	E	M	A	I	D	C
L	E	L	C	R	I	C	G	U
E	T	C	X	R	Y	O	H	B
L	E	I	T	R	A	N	T	E
L	S	Q	U	A	R	E	G	L
A	L	P	L	A	N	E	N	A
R	O	L	A	I	I	G	E	U
A	P	U	L	C	N	A	L	Q
P	E	S	M	M	E	T	R	E

Answers

Exercise 3:02A

1 a $2a$ b $\frac{x}{5}$

c $\frac{5a}{3}$ d $\frac{3m}{5}$

e $\frac{x+y}{4}$ f $\frac{5a-2b}{3}$

g $\frac{5}{a}$ h $\frac{8}{x}$

i $\frac{1}{y}$ j $\frac{8}{m}$

k $\frac{7a}{x}$ l $\frac{-x}{y}$

m $\frac{4}{n}$ n $\frac{1}{x}$

o $\frac{2a}{b}$ p $\frac{m}{x}$

2 a $\frac{8x}{15}$ b $\frac{7a}{10}$

c $\frac{y}{12}$ d $\frac{m}{4}$

e $\frac{7a}{6}$ f $\frac{13x}{6}$

g $\frac{n}{8}$ h $\frac{p}{2}$

i $\frac{3x+4y}{12}$ j $\frac{4a-9b}{6}$

k $\frac{6m+5n}{10}$ l $\frac{2k-63}{12}$

m $\frac{10}{3x}$ n $\frac{5}{6a}$

o $\frac{31}{10m}$ p $\frac{1}{8x}$

q $\frac{13a}{6x}$ r $\frac{-5x}{3m}$

s $\frac{13m}{4n}$ t $\frac{8x+3y}{12a}$

Answers

FUN SPOT 3:02

The mystery word is 'GRAM'.

Teaching strategies

Lowest common denominator (LCD)

Remind students that algebraic fractions cannot be added or subtracted if the denominator is not the same. Spend time demonstrating to students how to find common multiples.

One way to find the LCD is to look at tree diagrams and prime numbers. Design a tree diagram for finding the prime numbers in each denominator. Find the prime number that occurs the most in each denominator.

For example: LCD of $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{6}$

2 appears the most in 8 (3 times)

3 appears the most in 6 (1 time).

Multiply these numbers to give the LCD.

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{6}$

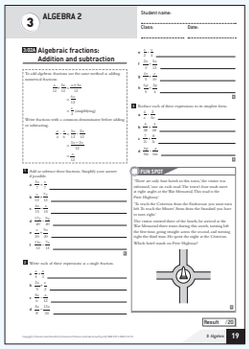
8 4 6

2×4 2×2 2×3

2×2

LCD → $2\times2\times2\times3 = 24$

Homework 3:02A



Answers

PREP QUIZ 3:02B

1 $\frac{3}{8}$ 2 $\frac{3}{10}$ 3 $\frac{1}{6}$ 4 $\frac{2}{3}$
5 2 6 $\frac{8}{15}$ 7 $30x$ 8 $6a^2$
9 $3a$ 10 $2a$

Lesson starter

Prep quiz 3:02B

Have students complete Prep quiz 3:02B as a lesson starter.

Teaching strategies

Copy, change and flip

Revise the copy, change and flip technique for division of fractions.

$$\frac{1}{2} \div \frac{3}{4}$$

copy ↓ change ↓ flip ↓

$$\frac{1}{2} \times \frac{4}{3}$$

3:02B Multiplication and division

PREP QUIZ 3:02B

Answer the following:

1 $\frac{1}{2} \times \frac{3}{4}$ 2 $\frac{2}{5} \times \frac{3}{4}$ 3 $\frac{4}{9} \times \frac{3}{8}$
4 $\frac{1}{2} \div \frac{3}{4}$ 5 $\frac{3}{5} \div \frac{3}{10}$ 6 $\frac{2}{3} \div \frac{5}{4}$

Simplify these expressions.

7 $5 \times 6x$ 8 $3a \times 2a$ 9 $15a \div 5$ 10 $12ab \div 6b$

When *multiplying*:
- cancel any common factors, then
- multiply the numerators together and multiply the denominators together.

When *dividing*:
- turn the second fraction upside down, then
- multiply as above (invert and multiply).

WORKED EXAMPLES

1 $\frac{2}{a} \times \frac{5}{b} = \frac{2 \times 5}{a \times b} = \frac{10}{ab}$

2 $\frac{5}{x} \times \frac{x}{10} = \frac{\cancel{5}^{1}}{\cancel{x}_{1}} \times \frac{\cancel{x}^{1}}{\cancel{10}_{2}} = \frac{1 \times 1}{1 \times 2} = \frac{1}{2}$

3 $\frac{3b}{2} \times \frac{4}{5b} = \frac{3\cancel{b}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{5\cancel{b}_{1}} = \frac{3 \times 2}{1 \times 5} = \frac{6}{5}$ or $1\frac{1}{5}$

4 $\frac{ab}{2} \div \frac{b}{5} = \frac{a\cancel{b}^{1}}{2} \times \frac{5}{\cancel{b}_{1}} = \frac{a \times 5}{2 \times 1} = \frac{5a}{2}$

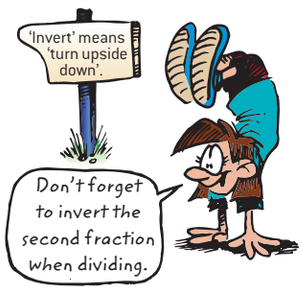
5 $\frac{8a}{3b} \div \frac{2a}{9b} = \frac{\cancel{8}^{4}\cancel{a}}{\cancel{3b}_{1}} \times \frac{\cancel{9}^{3}\cancel{b}}{\cancel{2a}_{1}} = \frac{4 \times 3}{1 \times 1} = 12$

6 $\frac{2a}{3b} \times \frac{b}{4c} \div \frac{10a}{9c} = \frac{4a}{3b} \times \frac{b}{2c} \times \frac{9c}{10a}$

$= \frac{\cancel{2}^{1}\cancel{4}\cancel{a}}{\cancel{3b}_{1}} \times \frac{\cancel{b}^{1}}{\cancel{2c}_{1}} \times \frac{\cancel{9}^{3}\cancel{c}}{\cancel{10}_{5}\cancel{a}}$

$= \frac{1 \times 1 \times 3}{1 \times 1 \times 5}$

$= \frac{3}{5}$



Teacher's notes

Exercise 3:02B

P Foundation worksheet 3:02B Simplifying algebraic fractions

1 Simplify these products.

a $\frac{x}{2} \times \frac{y}{3}$ b $\frac{a}{4} \times \frac{b}{3}$ c $\frac{m}{2} \times \frac{m}{5}$ d $\frac{a}{4} \times \frac{a}{10}$

e $\frac{3}{a} \times \frac{4}{m}$ f $\frac{2}{x} \times \frac{1}{y}$ g $\frac{1}{p} \times \frac{4}{p}$ h $\frac{1}{n} \times \frac{1}{3n}$

i $\frac{p}{q} \times \frac{x}{y}$ j $\frac{2}{a} \times \frac{a}{4}$ k $\frac{m}{5} \times \frac{10}{n}$ l $\frac{3x}{5} \times \frac{2}{9x}$

m $\frac{ab}{3} \times \frac{2}{b}$ n $\frac{x}{y} \times \frac{y}{x}$ o $\frac{6m}{5a} \times \frac{15a}{2m}$ p $\frac{8x}{5p} \times \frac{2a}{3x}$

2 Simplify these divisions.

a $\frac{m}{2} \div \frac{m}{4}$ b $\frac{n}{3} \div \frac{n}{5}$ c $\frac{5n}{3} \div \frac{2n}{9}$ d $\frac{x}{5} \div \frac{3x}{10}$

e $\frac{5}{a} \div \frac{2}{a}$ f $\frac{3}{2m} \div \frac{1}{3m}$ g $\frac{a}{b} \div \frac{2a}{b}$ h $\frac{3x}{5y} \div \frac{x}{10y}$

i $\frac{a}{b} \div \frac{x}{y}$ j $\frac{2p}{3q} \div \frac{8p}{9q}$ k $\frac{10k}{3n} \div \frac{2k}{9n}$ l $\frac{a}{2} \div \frac{a}{3}$

m $\frac{xy}{2} \div \frac{y}{4}$ n $\frac{b}{2} \div \frac{ab}{6}$ o $\frac{xy}{c} \div \frac{y}{cx}$ p $\frac{9a}{b} \div \frac{4a}{3b}$

3 Simplify these expressions.

a $\frac{a}{3} \div \frac{12}{5a}$ b $\frac{2}{p} \times \frac{p}{3}$ c $\frac{15}{x} \div 5$ d $3b \div \frac{6}{b}$

e $\frac{xy}{z} \times \frac{2z}{x}$ f $\frac{ab}{c} \div \frac{a}{c}$ g $\frac{9m}{2} \times \frac{4m}{3}$ h $\frac{2x}{y} \div \frac{x}{2y}$

i $\frac{4}{pq} \times \frac{p}{q}$ j $\frac{3}{a} \times \frac{2}{b}$ k $\frac{4ab}{x} \times \frac{xy}{2ac}$ l $\frac{9bc}{2a} \div \frac{6b}{4a}$

m $\frac{2}{x} \times \frac{x}{3} \times \frac{9}{4}$ n $\frac{b}{c} \times \frac{c}{a} \times \frac{a}{b}$ o $\frac{8bc}{3a} \times \frac{9a}{b} \times \frac{1}{4c}$ p $\frac{8}{a} \times \frac{2a}{15} \div \frac{8}{3}$

q $\frac{2m}{3n} \times \frac{5n}{6p} \div \frac{8m}{9p}$ r $\frac{6a}{15} \div \frac{3a}{10b} \times \frac{3}{4b}$ s $\frac{xy}{yz} \times \frac{xz}{ty} \times \frac{tz}{tx}$ t $\frac{2a}{3b} \div \frac{3a}{2b} \div \frac{4a}{9b}$

Algebra is important in the design and construction of buildings.

Teacher's notes

Answers

Exercise 3:02B

1 a $\frac{xy}{6}$ b $\frac{ab}{12}$ c $\frac{m^2}{10}$ d $\frac{a^2}{40}$

e $\frac{12}{am}$ f $\frac{2}{xy}$ g $\frac{4}{p^2}$ h $\frac{1}{3n^2}$

i $\frac{px}{qy}$ j $\frac{1}{2}$ k $\frac{2m}{n}$ l $\frac{2}{15}$

m $\frac{2a}{3}$ n 1 o 9 p $\frac{16a}{15p}$

2 a 2 b $\frac{5}{3}$ c $\frac{15}{2}$ d $\frac{2}{3}$

e $\frac{5}{2}$ f $\frac{9}{2}$ g $\frac{1}{2}$ h 6

i $\frac{ay}{bx}$ j $\frac{3}{4}$ k 15 l $\frac{3}{2}$

m $2x$ n $\frac{3}{a}$ o x^2 p $\frac{27}{4}$

3 a $\frac{5a^2}{36}$ b $\frac{2}{3}$ c $\frac{3}{x}$ d $\frac{b^2}{2}$

e $2y$ f b g $6m^2$ h 4

i $\frac{4}{q^2}$ j $\frac{6}{ab}$ k $\frac{2by}{c}$ l $3c$

m $\frac{3}{2}$ n 1 o 6 p $\frac{2}{5}$

q $\frac{5}{8}$ r 1 s $\frac{xz}{ty}$ t $\frac{b}{a}$

Class activities

Building algebraic fractions

Divide the class into groups of four. Each group must build 10 algebraic fractions. The rules are simple.

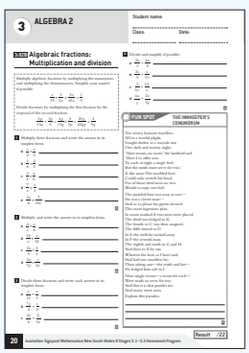
The answer must:

- contain at least one pronumeral
- be in the form of a fraction
- simplify from the given expression.

Once each group has made their fractions, they are passed to other groups to simplify.

[Understanding, Fluency]

Homework 3:02B



3:03 Content statements

Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate (ACMNA213) [Stage 5.2]

- expand algebraic expressions, including those involving terms with indices and/or negative coefficients, eg $-3x^2(5x^2 + 2x^4y)$
- expand algebraic expressions by removing grouping symbols and collecting like terms where applicable, eg expand and simplify $2y(y - 5) + 4(y - 5)$, $4x(3x + 2) - (x - 1)$

Answers

PREP QUIZ 3:03

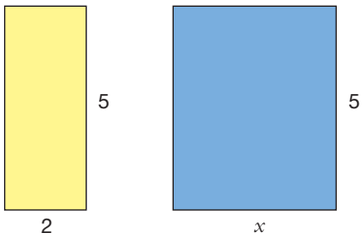
1 $10x$	**2** $3a^2$	**3** $6x + 8$
4 $x + 2$	**5** $5y^2 + 4y$	**6** $13 + 2a$
7 $3x - 21$	**8** $18 - 45y$	
9 $2a^2 + 6a$	**10** $-5x - 35$	

Lesson starter

Algebra tiles

Have students complete the following tasks.

1 Make a number of algebra tiles as shown below.



2 Work out the area of each algebra tile.

3 Using the algebra tiles, find ways to represent an area of:

a $5x + 10$

b $10x + 10$

c $5x + 20$

d $5x - 10$

Digital resources

eBook

- Foundation worksheet 3:03 Grouping symbols
- Challenge worksheet 3:03 Fractions and grouping symbols

ProductLink

- Grouping symbols (Drag-and-drop)

3:03 Simplifying expressions with grouping symbols

PREP QUIZ 3:03

Simplify:

1 $7x + 3x$	**2** $4a^2 - a^2$	**3** $4x + 3 + 2x + 5$
4 $2x + 7 - x - 5$	**5** $3y^2 + 5y + 2y^2 - y$	**6** $7 - 3a + 6 + 5a$

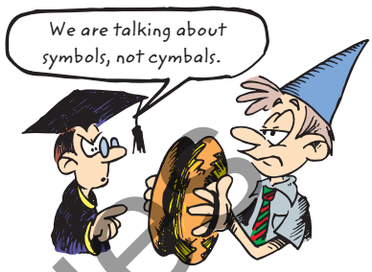
Expand:

7 $3(x - 7)$	**8** $9(2 - 5y)$	**9** $2a(a + 3)$	**10** $-5(x + 7)$

The two most commonly used grouping symbols are:

parentheses	( )
brackets	[ ]

$a(b \pm c) = ab \pm ac$



To expand an expression, such as $a(b + c)$, each term inside the grouping symbols is multiplied by the term outside the grouping symbols.

WORKED EXAMPLES

1 $p(p + 3) = p \times p + p \times 3$
$= p^2 + 3p$

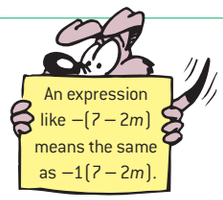
2 $3a(5 - 2a) = 3a \times 5 - 3a \times 2a$
$= 15a - 6a^2$

3 $-5(3x + 4) = (-5) \times 3x + (-5) \times 4$
$= -15x - 20$

4 $-(7 - 2m) = (-1) \times 7 - (-1) \times 2m$
$= -7 + 2m$

5 $x(x - 1) - x^2 + 5 = x^2 - x - x^2 + 5$
$= -x + 5$

6 $2a(a + b) - a(3a - 4b) = 2a^2 + 2ab - 3a^2 + 4ab$
$= 6ab - a^2$



Teaching strategies

Distributive law

Remind students of the distributive law.

$a(b + c) = a \times b + a \times c$

Use the distributive law to expand and simplify.

$3(3p + 2q) - 5(p - q)$
$= 3 \times 3p + 3 \times 2q + -5 \times p + -5 \times -q$
$= 9p + 6q - 5p + 5q$
$= (9p - 5p) + (6q + 5q)$
$= 4p + 11q$

Expanding grouping symbols

When expanding expressions that contain grouping symbols, follow these steps.

Step 1 Write the expression.

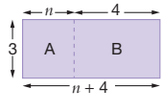
Step 2 Work from left to right to expand the grouping symbols.

Step 3 Collect like terms.

Exercise 3:03

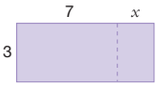
P Foundation worksheet 3:03 Grouping symbols
Challenge worksheet 3:03 Fractions and grouping symbols

1 The area of rectangle A = $3 \times n = 3n$
The area of rectangle B = $3 \times 4 = 12$
The area of the combined rectangle = $3(n + 4)$
$\therefore\ 3(n + 4) = 3n + 12$

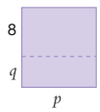


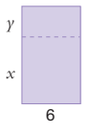
Following the example above, write the area of each of the following rectangles in two ways.

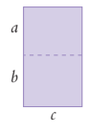
a 

b 

c 

d 

e 

f 

2 Expand:

a $2(x + 3)$	b $3(a + 5)$	c $5(x - 1)$	d $7(m - 3)$
e $2(3a + 2b)$	f $5(x + y)$	g $7(3x - 5y)$	h $6(7m - 8n)$
i $x(x + 7)$	j $a(a - 1)$	k $m(m + 10)$	l $n(n - 3)$
m $3a(2a - 1)$	n $9x(2x + 7)$	o $8p(2 - 5p)$	p $7q(3 + 2q)$
q $x(a + b)$	r $y(2y + x)$	s $2m(m + n)$	t $5a(2a + 3b)$

3 Expand by removing the parentheses.

a $-2(x + 3)$	b $-3(a + 5)$	c $-2(y - 1)$	d $-5(p - 3)$
e $-7(3a + 2)$	f $-5(2x - 1)$	g $-7(3 + 2m)$	h $-4(7 - x)$
i $-(a + 1)$	j $-(3x + 7)$	k $-(8 - 2p)$	l $-(3a + 2b)$
m $-x(x + 10)$	n $-y(5 - y)$	o $-3x(2x + 7)$	p $-10n(8m - n)$

4 Simplify:

a $2(a + 3) + 5a + 2$	b $3(x + 5) + 7x - 8$	c $5(y - 2) + 3y + 7$
d $4(a - 1) + 6a - 5$	e $3(p + 2) - 2p + 4$	f $10(m + 3) - 11m - 15$
g $5a + 6 + 2(a + 7)$	h $2x + 7 + 5(x - 1)$	i $7n - 4 + 3(n - 1)$
j $4h - 1 + 7(h + 2)$	k $6x + 2(x + 1) + 5$	l $4y + 6(y + 2) - 10$
m $3a + 10 - 2(a + 1)$	n $10m + 4 - 5(m + 4)$	o $6 - 2(y - 4) + 4y$
p $20 - 4(x - 2) + 5x$	q $5x + 7 + 2(2x + 7)$	r $4(3a + 1) - 10a + 2$
s $10m + 6 - 3(2m - 1)$	t $8x - 3(1 - 2x) + 10$	

5 Simplify each expression by expanding the grouping symbols and then collecting like terms.

a $3(x + 2) + 2(x + 1)$	b $5(y + 2) + 3(y + 4)$	c $2(a + 1) + 5(a - 1)$
d $8(m - 3) + 5(m + 2)$	e $4(3x + 2) + 5(x - 4)$	f $6(x + 7) + 2(2x - 1)$
g $5(x + 7) - 3(x + 4)$	h $6(m + 1) - 3(m + 2)$	i $9(a + 5) - 7(a - 3)$
j $5(n - 5) - 3(n + 7)$	k $x(x + 3) + 3(x + 1)$	l $a(a + 3) + 7(a - 3)$
m $m(m + 3) - 4(m + 3)$	n $t(t - 5) - 4(t - 5)$	o $a(a + 2b) + a(2a + b)$
p $x(x + y) + y(x + y)$		

Class activities

Find my number bingo

Students will need: a 3×3 grid in which to write the following nine numbers in any order:

$-20, 13, -39, 4, -14, -57, 6, 12, -15$

Each question is shown to students one at a time within a short interval.

Students find the missing number and then cross that number off their bingo sheet. The winner is the first to cross off three numbers in a row, column or diagonal.

1 $x(6x^2 + 7) = \square x^3 + 7x$

2 $2x(5x^2 + 6) = 10x^3 + \square x$

3 $-5x(3x - 7) = \square x^2 + 35x$

4 $2(x + 4) + 4(x - 7) = 6x + \square$

5 $7(x + 4) + 3(2x - 5) = \square x + 13$

6 $5(3x + 1) + 4(3x - 11) = 27x + \square$

7 $5(5x - 4) - 4(3x - 6) = 13x + \square$

8 $2(11 - 15x) - 9(3x + 4) = \square x - \square$

Answers:

1 6 **2** 12 **3** −15 **4** −20
5 13 **6** −39 **7** 4 **8** −57, 14

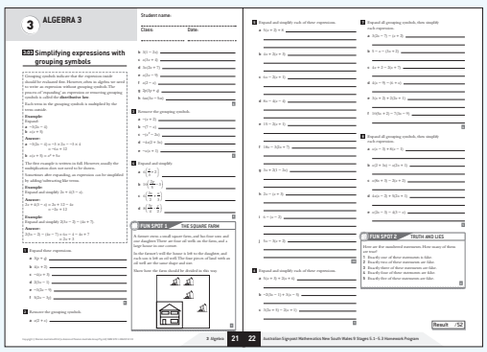
[Understanding, Fluency]

Answers

Exercise 3:03

1 a $5a + 35, 5(a + 7)$	b $21 + 3x, 3(7 + x)$	
c $7m + 7n, 7(m + n)$	d $pq + 8p, p(q + 8)$	
e $6x + 6y, 6(x + y)$	f $bc + ac, c(b + a)$	
2 a $2x + 6$	b $3a + 15$	
c $5x - 5$	d $7m - 21$	
e $6a + 4b$	f $5x + 5y$	
g $21x - 35y$	h $42m - 48n$	
i $x^2 + 7x$	j $a^2 - a$	
k $m^2 + 10m$	l $n^2 - 3n$	
m $6a^2 - 3a$	n $18x^2 + 63x$	
o $16p - 40p^2$	p $21q + 14q^2$	
q $ax + bx$	r $2y^2 + xy$	
s $2m^2 + 2mn$	t $10a^2 + 15ab$	
3 a $-2x - 6$	b $-3a - 15$	
c $-2y + 2$	d $-5p + 15$	
e $-21a - 14$	f $-10x + 5$	
g $-21 - 14m$	h $-28 + 4x$	
i $-a - 1$	j $-3x - 7$	
k $-8 + 2p$	l $-3a - 2b$	
m $-x^2 - 10x$	n $-5y + y^2$	
o $-6x^2 - 21x$	p $-80mn + 10n^2$	
4 a $7a + 8$	b $10x + 7$	
c $8y - 3$	d $10a - 9$	
e $p + 10$	f $15 - m$	
g $7a + 20$	h $7x + 2$	
i $10n - 7$	j $11h + 13$	
k $8x + 7$	l $10y + 2$	
m $a + 8$	n $5m - 16$	
o $2y + 14$	p $x + 28$	
q $9x + 21$	r $2a + 6$	
s $4m + 9$	t $14x + 7$	
5 a $5x + 8$	b $8y + 22$	
c $7a - 3$	d $13m - 14$	
e $17x - 12$	f $10x + 40$	
g $2x + 23$	h $3m$	
i $2a + 66$	j $2n - 46$	
k $x^2 + 6x + 3$	l $a^2 + 10a - 21$	
m $m^2 - m - 12$	n $t^2 - 9t + 20$	
o $3a^2 + 3ab$	p $x^2 + 2xy + y^2$	

Homework 3:03



Class activities

Expand and simplify matching pairs

Make a set of cards as shown below, and ask students to match the question with the answer.

Expand and simplify $2x(3y - 2) - 5$	$8x^2 - 11x - 6$
$24x^2 - 9x + 21$	Expand and simplify $12x - (5 - 4x)$
Expand and simplify $2x(5 - 4x) - 3(2 + 7x)$	Expand and simplify $12x(2x - 1) + 3(x + 7)$
Expand and simplify $8x - 5(2x^2 - 7x)$	$-10x^2 + 43x$
$16x - 5$	$6xy - 4x - 5$

Answers:

Expand and simplify $2x(3x - 2) - 5$	$6x^2 - 4x - 5$
Expand and simplify $12x(2x - 1) + 3(x + 7)$	$24x^2 - 9x + 21$
Expand and simplify $12x - (5 - 4x)$	$16x - 5$
Expand and simplify $2x(5 - 4x) - 3(2 + 7x)$	$8x^2 - 11x - 6$
Expand and simplify $8x - 5(2x^2 - 7x)$	$-10x^2 + 43x$

Extension

Extension expansions

Show how you could use an area model to expand each of the following expressions.

1 $(a + 5)(b + 2)$

2 $(a + 5)(b - 2)$

3 $(3x + 5y)(2x + 7y)$

4 $(6y - 7z)(2y - 3z)$

Answers:

	a	5
b	ab	$5b$
2	$2a$	10

1 $ab + 2a + 5b + 10$

2 $ab - 2a + 5b - 10$

3 $6x^2 + 31xy + 35y^2$

4 $12y^2 - 32yz + 21z^2$

[Understanding, Fluency]

FUN SPOT 3:03 — WHAT IS TAKEN OFF LAST BEFORE YOU GET INTO BED?

Work out the answer to each part and write the letter for that part in the box that is above the correct answer.

Write the expression that is:

H 2 more than x — L twice x

K half of x — N 2 less than x

R the square of x

Find the value of $u + 10t$ if:

R $u = 12, t = 4$ — R $u = -10, t = 2$

Simplify:

T $x \div x$ — T $11x - x$ — T $x + x + x$ — F $x \times x \times 2$

F $x^2 - x - x$ — F $7x + 3y - x + 3y$ — F $3x \times 2y$ — A $-2x \times -3x$

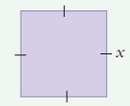
A $12x \div 2$ — Y $12m \div 4m$ — O $14 - 3x + 6$ — E $\frac{x}{2} + \frac{x}{2}$

O $\frac{2x}{5} - \frac{x}{10}$ — O $\frac{5}{x} \times \frac{x}{10}$ — O $x - \frac{x}{10}$ — U $x \div \frac{1}{8}$

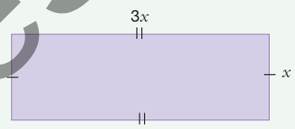
E $\frac{x}{3} \div \frac{x}{6}$ — E $\frac{x}{2} \div 2$

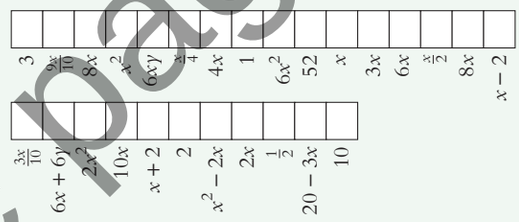
Write an expression for the perimeter of:

E



E





Answers

FUN SPOT 3:03

YOUR FEET ARE TAKEN OFF THE FLOOR

3:04 Further algebraic fractions

PREP QUIZ 3:04

Expand:

1 $3(x + 5)$ **2** $5(m - 7)$ **3** $2(6y + 1)$ **4** $9(2p - 3q)$

Find the lowest common multiple (LCM) of:

5 4 and 5 **6** 4 and 6 **7** 4 and 8

Simplify:

8 $\frac{x}{4} + \frac{x}{5}$ **9** $\frac{a}{4} - \frac{a}{6}$ **10** $\frac{m}{8} + \frac{n}{4}$

In Section 3:02A all of the fractions had a single term in the numerator. If there is more than one term in the numerator we use the skills met in the last section for expanding grouping symbols.

WORKED EXAMPLES

Simplify the following:

1 $\frac{x+1}{2} + \frac{x+4}{3} = \frac{(x+1)\times 3}{2\times 3} + \frac{(x+4)\times 2}{3\times 2}$

$= \frac{3(x+1)}{6} + \frac{2(x+4)}{6}$

$= \frac{3x+3}{6} + \frac{2x+8}{6}$

$= \frac{5x+11}{6}$

2 $\frac{2a+5}{3} - \frac{3a}{5} = \frac{(2a+5)\times 5}{3\times 5} - \frac{3a\times 3}{5\times 3}$

$= \frac{5(2a+5)}{15} - \frac{9a}{15}$

$= \frac{10a+25}{15} - \frac{9a}{15}$

$= \frac{a+25}{15}$

LCM of 3 and 5 is 15

3 $\frac{2x+5}{6} + \frac{x-4}{3} = \frac{2x+5}{6} + \frac{2(x-4)}{6}$

$= \frac{2x+5}{6} + \frac{2x-8}{6}$

$= \frac{4x-3}{6}$

4 $\frac{m}{3} - \frac{m+1}{4} = \frac{4m}{12} - \frac{3(m+1)}{12}$

$= \frac{4m-3(m+1)}{12}$

$= \frac{4m-3m-3}{12}$

$= \frac{m-3}{12}$

Note the minus sign!

Teacher's notes

3:04 Content statements

Add and subtract algebraic fractions with numerical denominators, including those with binomial numerators (NSW) [Stage 5.3§]

- add and subtract algebraic fractions, including those with binomial numerators,

eg $\frac{2x+5}{6} + \frac{x-4}{3}, \frac{x}{3} - \frac{x+1}{5}$

Answers

PREP QUIZ 3:04

1 $3x + 15$ **2** $5m - 35$ **3** $12y + 2$

4 $18p - 27q$ **5** 20 **6** 12

7 8 **8** $\frac{9x}{20}$ **9** $\frac{a}{12}$

10 $\frac{m+2n}{8}$

Lesson starter

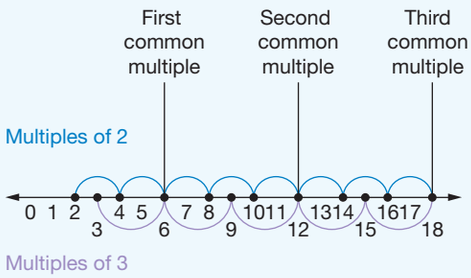
Prep quiz 3:04

Have students complete Prep quiz 3:04 as a lesson starter.

Teaching strategies

Finding LCM

Have students develop a number line as a technique to find the LCM for algebraic fractions.



Common mistakes

Ask students to find the common mistake in the following solution.

$\frac{2x+5}{3} - \frac{4x-3}{4}$

$= \frac{8x+5}{12} - \frac{12x-3}{12}$

$= \frac{-4x+2}{12}$

Discuss with students the following points:

- You must use the distributive law correctly and multiply each term in the numerator by the factor required to give the LCD.
- Make sure you correctly multiply the expression by the negative number.

Answers

Exercise 3:04

1 a $\frac{5x+12}{6}$ b $\frac{7a+15}{10}$ c $\frac{13n-7}{21}$
d $\frac{8x+17}{15}$ e $\frac{12m+13}{35}$ f $\frac{p+4}{12}$

2 a $\frac{3a+4}{4}$ b $\frac{3w+2}{10}$ c $\frac{7d-2}{6}$
d $\frac{5x+13}{12}$ e $\frac{7m+2}{24}$ f $\frac{q+3}{18}$

3 a $\frac{8x+17}{6}$ b $\frac{22a+25}{20}$ c $\frac{32n+5}{12}$
d $\frac{27t-17}{15}$ e $\frac{37u-30}{24}$ f $\frac{65q+8}{90}$
g $\frac{4a+5}{6}$ h $\frac{5n+3}{8}$ i $\frac{19g}{9}$
j $\frac{25y-7}{24}$ k $\frac{8-2x}{15}$ l $\frac{13-14a}{40}$

4 a $\frac{x-2}{6}$ b $\frac{y-15}{10}$ c $\frac{2n-1}{15}$
d $\frac{x+29}{30}$ e $\frac{2m+8}{35}$ f $\frac{5p+8}{12}$
g $\frac{1}{6}$ h $\frac{t+4}{8}$ i $\frac{11w}{9}$
j $\frac{19-2y}{24}$ k $\frac{-1-x}{30}$ l $\frac{7+2z}{40}$

5 a $\frac{31x+91}{30}$ b $\frac{86a+35}{60}$ c $\frac{14m+13}{6}$
d $\frac{12y+29}{20}$ e $\frac{5x-3}{4}$ f $\frac{4z+13}{20}$

Extension

Extra challenge

Simplify these expressions.

1 $\frac{2x+3}{x-4}+\frac{x+7}{x-4}$

2 $\frac{3x-5}{x+1}-\frac{2x+4}{x+1}$

3 $\frac{x-1}{x-3}-\frac{x-2}{3-x}$

4 $\frac{x-1}{2x-5}+\frac{3-x}{5-2x}$

Answers:

1 $\frac{3x+10}{x-4}$

2 $\frac{x-9}{x+1}$

3 $\frac{1}{x-3}$

4 $\frac{2x-4}{2x-5}$

Exercise 3:04

1 Simplify the following.

a $\frac{x+4}{2}+\frac{x}{3}$ b $\frac{a}{5}+\frac{a+3}{2}$ c $\frac{n-1}{3}+\frac{2n}{7}$
d $\frac{x+4}{5}+\frac{x+1}{3}$ e $\frac{m+4}{5}+\frac{m-3}{7}$ f $\frac{p+1}{3}-\frac{p}{4}$

2 Simplify:

a $\frac{a+4}{4}+\frac{a}{2}$ b $\frac{w}{5}+\frac{w+2}{10}$ c $\frac{d-1}{3}+\frac{5d}{6}$
d $\frac{x+3}{4}+\frac{x+2}{6}$ e $\frac{m+2}{8}+\frac{m-1}{6}$ f $\frac{q+1}{6}-\frac{q}{9}$

3 Simplify these expressions.

a $\frac{2x+3}{2}+\frac{x+4}{3}$ b $\frac{3a+5}{5}+\frac{2a+1}{4}$ c $\frac{5n-1}{3}+\frac{4n+3}{4}$
d $\frac{4t-4}{5}+\frac{3t-1}{3}$ e $\frac{7u-2}{8}+\frac{2u-3}{3}$ f $\frac{5q+2}{10}+\frac{2q-1}{9}$
g $\frac{2a+3}{6}+\frac{a+1}{3}$ h $\frac{3n+5}{8}+\frac{n-1}{4}$ i $\frac{5g-1}{3}+\frac{4g+3}{9}$
j $\frac{4y-1}{6}+\frac{3y-1}{8}$ k $\frac{5-2x}{6}+\frac{2x-3}{10}$ l $\frac{1-2a}{8}+\frac{2-a}{10}$

4 Simplify these fractions.

a $\frac{x}{2}-\frac{x+1}{3}$ b $\frac{3y}{5}-\frac{y+3}{2}$ c $\frac{n+1}{3}-\frac{n+2}{5}$
d $\frac{x+4}{5}-\frac{x-1}{6}$ e $\frac{m-1}{5}-\frac{m-3}{7}$ f $\frac{2p-1}{3}-\frac{p-4}{4}$
g $\frac{2q+3}{6}-\frac{q+1}{3}$ h $\frac{3t+2}{8}-\frac{t-1}{4}$ i $\frac{5w-2}{3}-\frac{4w-6}{9}$
j $\frac{4+y}{6}-\frac{2y-1}{8}$ k $\frac{1-2x}{6}-\frac{2-3x}{10}$ l $\frac{3-2z}{8}-\frac{2-3z}{10}$

5 Simplify these expressions.

a $\frac{x+3}{2}+\frac{x+4}{3}+\frac{x+1}{5}$ b $\frac{3a+5}{5}+\frac{2a+1}{4}+\frac{a-2}{3}$ c $\frac{2m+3}{3}+\frac{m+4}{6}+\frac{3m+1}{2}$
d $\frac{3y+1}{5}+\frac{2y-1}{4}-\frac{y-3}{2}$ e $\frac{5x-1}{4}-\frac{x+2}{3}+\frac{2x+1}{6}$ f $\frac{3z+4}{5}-\frac{2z-1}{4}-\frac{4-z}{10}$

CHALLENGE 3:04 THESE ARE A FRACTION TRICKIER!

Consider how you simplified expressions in Exercise 3:02A that involved pronumerals in the denominator, and attempt to simplify the following expressions.

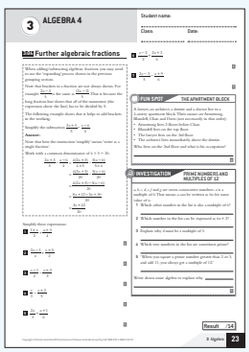
1 $\frac{x+4}{2x}+\frac{x+1}{3}$ 2 $\frac{x+4}{2x}+\frac{x+1}{3x}$ 3 $\frac{a+5}{5x}+\frac{a-2}{2x}$ 4 $\frac{x+5}{5x}+\frac{a+1}{2a}$
5 $\frac{x-4}{3x}-\frac{x-6}{5x}$ 6 $\frac{3n+4}{4n}-\frac{5-n}{6n}$ 7 $\frac{a+b}{a}+\frac{a-b}{b}$ 8 $\frac{3x-5y}{2x}-\frac{2y+x}{3y}$

Answers

CHALLENGE 3:04

1 $\frac{2x^2+5x+12}{6x}$ 2 $\frac{5x+14}{6x}$
3 $\frac{7a}{10x}$ 4 $\frac{10a+7ax+5x}{10ax}$
5 $\frac{2x-2}{15x}$ 6 $\frac{7n+2}{12n}$
7 $\frac{a^2+b^2}{ab}$ 8 $\frac{5xy-2x^2-15y^2}{6xy}$

Homework 3:04


3 ALGEBRA 4

Student name:

Class: Date:

Further algebraic fractions

FUN SPOT THE APARTMENT BLOCK

INVESTIGATION PRIME NUMBERS AND MULTIPLES OF 12

Result /14

3:05 Factorising using common factors

PREP QUIZ 3:05

Expand:

1 $2(x + 5)$ **2** $x(4x - 1)$

3 $3a(2a + 7)$ **4** $2m(m + 3n)$

Write the factors of:

5 12 **6** 20 **7** 30

Write the highest common factor (HCF) of:

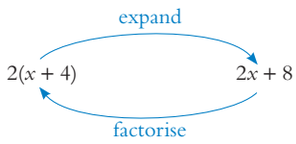
8 12 and 20 **9** 12 and 30 **10** 20 and 30

A factor of a given number is a number that divides into the given number with no remainder.
e.g. {1, 2, 3, 6, 9, 18} is the set of factors of 18.

To factorise an algebraic expression we must determine the highest common factor (HCF) of the terms and insert grouping symbols, usually parentheses.

If we expand the expression $2(x + 4)$, we obtain $2x + 8$. To factorise $2x + 8$ we simply reverse this procedure. The highest common factor of $2x$ and 8 is 2, so 2 is written outside the parentheses. The remainder is written inside the parentheses.

$2x + 8 = 2(x + 4)$



WORKED EXAMPLES

1 $2x + 6 = 2 \times x + 2 \times 3$ (HCF is 2)
$= 2(x + 3)$

2 $10x + 15y = 5 \times 2x + 5 \times 3y$ (HCF is 5)
$= 5(2x + 3y)$

3 $2mn + m = m \times 2n + m \times 1$ (HCF is m)
$= m(2n + 1)$

4 $9ax + 6a = 3a \times 3x + 3a \times 2$ (HCF is $3a$)
$= 3a(3x + 2)$

5 $5x^2 - 10xy = 5x \times x - 5x \times 2y$ (HCF is $5x$)
$= 5x(x - 2y)$

6 $-4a - 8 = -4 \times a + -4 \times 2$ (HCF is -4)
$= -4(a + 2)$

7 $-a^2 + 3a = -a \times a - (-a) \times 3$ (HCF is $-a$)
$= -a(a - 3)$

8 $4xy + 6x^2 - 2xz$ (HCF is $2x$)
$= 2x \times 2y + 2x \times 3x - 2x \times z$
$= 2x(2y + 3x - z)$

$ab + ac = a(b + c)$ and $ab - ac = a(b - c)$

Teaching strategies

My factorising steps

When factorising, check for numerical common factors and algebraic common factors.

Step 1 Find the common factors.

Step 2 Find the highest common factors (HCF).

Step 3 Factorise $ab + ac = a(b + c)$.

Step 4 Check your factorisation by multiplying the factors and comparing the answer with the original expression.

Remember: $a(b + c) = ab + ac$

Negative common factors

When we have common negative factors, the common factor -1 is used to factorise the expression in addition to the HCF. For example, when factorising $-9a - 12$, the common factors of -9 and -12 are -3, 3, -1 and 1.

The HCF of 3 and common factor of -1 are used to factorise the expression.

$-9a - 12 = -3(3a + 4)$

3:05 Content statements

Factorise algebraic expressions by taking out a common algebraic factor (ACMNA230) [Stage 5.2]

- factorise algebraic expressions, including those involving indices, by determining common factors, eg factorise $3x^2 - 6x$, $14ab + 12a^2$, $21xy - 3x + 9x^2$, $15p^2q^3 - 12pq^4$

Answers

PREP QUIZ 3:05

1 $2x + 10$
2 $4x^2 - x$
3 $6a^2 + 21a$
4 $2m^2 + 6mn$
5 {1, 2, 3, 4, 6, 12}
6 {1, 2, 4, 5, 10, 20}
7 {1, 2, 3, 5, 6, 10, 15, 30}
8 4
9 6
10 10

Lesson starter

Find my common factor

Ask students to find common factors of various numbers and pronumerals.

1 List all of the factors of 20.

2 List all of the factors of 50.

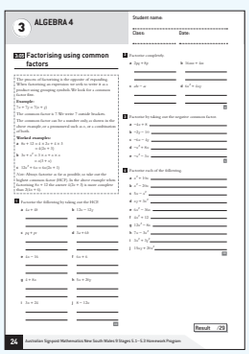
3 What factors are common to 20 and 50?

4 What is the highest common factor of 20 and 50?

5 What factors are common to $30x^2$ and $40x^3$?

6 What are the highest common factors of $30x^2$ and $40x^3$?

Homework 3:05



Digital resources

eBook

- Foundation worksheet 3:05 Common factors

Answers

Exercise 3:05

1 a $4(x+2)$ b $6(a+3)$ c $4(2a-3)$
d $5(x+2y)$ e $7(3x-2y)$ f $3(4pq-5x)$
g $n(m-p)$ h $a(b-c)$ i $2a(x+2y)$
j $y(y-5)$ k $at(5-3a)$ l $mn(7-n)$

2 a $2(x+5)$ b $2(3a+2)$ c $7(y+3)$
d $4(7+x)$ e $3(9-y)$ f $6(4x+1)$
g $9(x-5)$ h $4(4-3a)$ i $3(3x+y)$
j $5(a+2b)$ k $5(3m-4n)$ l $2(2b-3a)$
m $m(p+n)$ n $a(x+y)$ o $x(x+y)$
p $p(p-q)$ q $a(p+3)$ r $x(5+a)$
s $m(4-n)$ t $t(x-1)$

3 a $3a(x+2y)$ b $5m(n-2p)$ c $2b(2a-3c)$
d $3q(3p-2r)$ e $5x(x-2y)$ f $3a(b+2a)$
g $2m(5m-2n)$ h $4x(3x+y)$ i $bc(a+d)$
j $pq(a-b)$ k $xy(z+1)$ l $mn(1-p)$
m $xa(x-y)$ n $5x(a-2y)$ o $ap(a-5)$
p $xy(y+z)$ q $5b(2a-3c)$ r $xy(5x-3y)$
s $ap(p-a)$ t $ab(5-ab)$

4 a $-2(a+3)$ b $-5(x+3)$ c $-4(2m+3)$
d $-5(2x+1)$ e $-4(2x-1)$ f $-3(n-3)$
g $-7(y-5)$ h $-2(3a-2)$ i $-x(x+3)$
j $-m(m+1)$ k $-x(3x-2)$ l $-5y(y-2)$
m $-p(4+p)$ n $-x(3+2x)$ o $-m(1-7m)$
p $-2a(2-9a)$

5 a $a(b+c+d)$ b $x(3+y+z)$
c $m(m-3+n)$ d $a(7-b+a)$
e $p(p+q-5)$ f $2(x+2y-3z)$
g $5(2a-b+3c)$ h $3(3x^2+2x-4)$
i $2(4-2x+3x^2)$ j $5(5+3y-4y^2)$
k $x(xy-3y+1)$ l $2a(b-2c+5)$
m $3x(x+2y-3)$ n $xy(x+1+y)$
o $ab(ab+3a+2b)$ p $mn(1+4m-8n)$

6 a $(a+2)(a+3)$ b $(m+2)(m+4)$
c $(x-1)(x+5)$ d $(b+1)(b-5)$
e $(y-2)(7-y)$ f $(t-7)(t-9)$
g $(2m-3)(4+3m)$ h $(7x+1)(2x-5)$
i $(a+3)(x-1)$ j $(2y-1)(y-1)$
k $(p-3)(p-3)$ l $(5x+3)(1-x)$

Exercise 3:05

P Foundation worksheet 3:05 Common factors

1 Complete the following.

a $4x+8=4(\quad)$ b $6a+18=6(\quad)$ c $8a-12=4(\quad)$
d $5x+10y=5(\quad)$ e $21x-14y=7(\quad)$ f $12pq-15x=3(\quad)$
g $mn-np=n(\quad)$ h $ab-ac=a(\quad)$ i $2ax+4ay=2a(\quad)$
j $y^2-5y=y(\quad)$ k $5at-3a^2t=at(\quad)$ l $7mn-mn^2=mn(\quad)$

2 Factorise the following by taking out the HCF.

a $2x+10$ b $6a+4$ c $7y+21$ d $28+4x$
e $27-3y$ f $24x+6$ g $9x-45$ h $16-12a$
i $9x+3y$ j $5a+10b$ k $15m-20n$ l $4b-6a$
m $mp+mn$ n $ax+ay$ o x^2+xy p p^2-pq
q $ap+3a$ r $5x+ax$ s $4m-mn$ t $xt-t$

3 Factorise completely.

a $3ax+6ay$ b $5mn-10mp$ c $4ab-6bc$ d $9pq-6qr$
e $5x^2-10xy$ f $3ab+6a^2$ g $10m^2-4mn$ h $12x^2+4xy$
i $abc+bcd$ j $apq-bpq$ k $xyz+xy$ l $mn-mnp$
m x^2a-xay n $5ax-10xy$ o a^2p-5ap p xy^2+xyz
q $10ab-15bc$ r $5x^2y-3xy^2$ s ap^2-a^2p t $5ab-a^2b^2$

4 Factorise the following by taking out the negative common factor.

a $-2a-6$ b $-5x-15$ c $-8m-12$ d $-10x-5$
e $-8x+4$ f $-3n+9$ g $-7y+35$ h $-6a+4$
i $-x^2-3x$ j $-m^2-m$ k $-3x^2+2x$ l $-5y^2+10y$
m $-4p-p^2$ n $-3x-2x^2$ o $-m+7m^2$ p $-4a+18a^2$

5 Factorise each of the following.

a $ab+ac+ad$ b $3x+xy+xz$ c $m^2-3m+mn$ d $7a-ab+a^2$
e $p^2+pq-5p$ f $2x+4y-6z$ g $10a-5b+15c$ h $9x^2+6x-12$
i $8-4x+6x^2$ j $25+15y-20y^2$ k $x^2y-3xy+x$ l $2ab-4ac+10a$
m $3x^2+6xy-9x$ n $x^2y+xy+xy^2$ o $a^2b^2+3a^2b+2ab^2$ p $mn+4m^2n-8mn^2$

6 Examine this example.

If $3(x+2)-a(x+2)$ is to be factorised then $(x+2)$ is common to both terms, so it can be taken out as a common factor:

$3(x+2)-a(x+2)=(x+2)(3-a)$

Similarly:

$x(a+1)+(a+1)=x(a+1)+1(a+1)$
$=(a+1)(x+1)$

Note:
$(a+1)(x+1)=(x+1)(a+1)$

Now factorise these similar types.

a $a(a+2)+3(a+2)$ b $m(m+2)+4(m+2)$ c $x(x-1)+5(x-1)$
d $b(b+1)-5(b+1)$ e $7(y-2)-y(y-2)$ f $t(t-7)-9(t-7)$
g $4(2m-3)+3m(2m-3)$ h $2x(7x+1)-5(7x+1)$ i $x(a+3)-(a+3)$
j $y(2y-1)-(2y-1)$ k $p(p-3)-3(p-3)$ l $(5x+3)-x(5x+3)$

Class activities

My observations

Complete the following table.

Expanded form	My observation	Factorised expression
$16-8y$	Common factors: 1, 2, 4, 8 HCF is 8	$8(2-y)$
$14y-49$		
$15xy-3x^2y^2$		
$8a^2+12a$		
$-12x^2-24x$		
$d(2d-3)+3(2d-3)$		

Answers:

Expanded form	My observation	Factorised expression
$16-8y$	Common factors: 1, 2, 4, 8 HCF is 8	$8(2-y)$
$14y-49$	Common factors: 1, 7 HCF is 7	$7(2y-7)$
$15x^2y-3x^2y^2$	Common factors: 1, 3, x^2, y HCF is $3x^2y$	$3x^2y(5-y)$
$8a^2+12a$	Common factors: 1, 2, 4, a HCF is $4a$	$4a(2a-3)$
$-12x^2-24x$	Common factors: 1, −1, 2, 4, 6, 12, x HCF is $-12x$.	$-12x(x+2)$
$d(2d-3)+3(2d-3)$	Common factor: $(2d-3)$ HCF is $(2d-3)$	$(2d-3)(d+3)$

[Fluency]

3:06 Binomial products

PREP QUIZ 3:06

Simplify:

1 $5x + 7x$ 2 $2a - a$ 3 $x^2 + 3x - 5x + 3$

Expand:

4 $2(x + 5)$ 5 $x(x - 2)$ 6 $-3(a + 1)$ 7 $-y(5 - y)$

Expand and simplify:

8 $x(x + 1) + 3(x + 1)$ 9 $5(a + 5) - a(a + 5)$ 10 $2x(3x - 2) - 5(3x + 2)$

A binomial expression is one that contains two terms, such as $2x - 7$ or $a + b$.
A binomial product is the product of two such expressions, such as $(2x + 7)(a + 5)$.

Long multiplication is like a binomial product.

$26 \times 19 = (20 + 6) \times (10 + 9)$
$= 20(10 + 9) + 6(10 + 9)$
$= [20 \times 10] + [20 \times 9] + [6 \times 10] + [6 \times 9]$
$= 200 + 180 + 60 + 54$
$= 494$

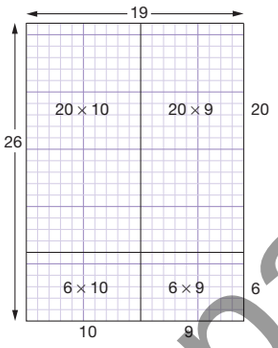
Each part of one number must multiply each part of the other.

$(20 + 6)\quad(10 + 9)$

As you can see, the products form a 'face'.

	19
×	26
	54
	60
	180
	200
	494

26 × 19 by areas

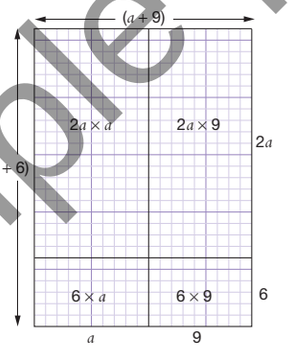


Multiplying binomial expressions

The expansion of binomial products may also be demonstrated by considering the area of a rectangle. This rectangle has dimensions $(2a + 6)$ and $(a + 9)$.

- The area of the whole rectangle must be equal to the sum of the four smaller areas.
- Area $= (2a + 6)(a + 9)$
  $= 2a(a + 9) + 6(a + 9)$
  $= 2a^2 + 18a + 6a + 54$
  $= 2a^2 + 24a + 54$
- We can see that the product of two binomials yields four terms. Often two of these may be added together to simplify the answer.

$(2a + 6)(a + 9)$ by areas



3:06 Content statements

Expand binomial products and factorise monic quadratic expressions using a variety of strategies (ACMNA233) [Stage 5.2]

- use algebraic methods to expand binomial products, eg $(x + 2)(x - 3)$, $(4a - 1)(3a + 2)$

Expand binomial products using a variety of strategies (ACMNA233) [Stage 5.3§]

- simplify a variety of expressions involving binomial products, eg $(3x + 1)(2 - x) + 2x + 4$, $(x - y)^2 - (x + y)^2$

Answers

PREP QUIZ 3:06

1 $12x$ 2 a 3 $x^2 - 2x + 3$
4 $2x + 10$ 5 $x^2 - 2x$ 6 $-3a - 3$
7 $-5y + y^2$ 8 $x^2 + 4x + 3$ 9 $25 - a^2$
10 $6x^2 - 19x - 10$

Lesson starter

Long multiplication and binomial products

Demonstrate to students how long multiplication (two digits) is like a binomial product, using the following example. Have each student create a different long multiplication problem for others to solve.

45 × 37	30	7
40	40 × 30 1200	40 × 7 280
5	5 × 30 150	5 × 7 35

1200
280
150
+ 35
1665

Teaching strategies

Using tables for binomial expressions

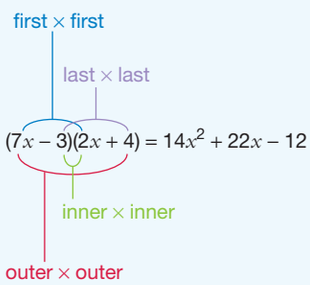
Demonstrate to students how to use a table format to expand binomial expressions.

$(x + 3)(x - 2)$	x	-2
x	$x \times x$ x^2	$x \times -2$ $-2x$
3	$3 \times x$ $3x$	3×-2 -6

$x^2 + \underbrace{3x + (-2x)}_{x} + (-6) = x^2 + x - 6$

FOIL (First, Outer, Inner, Last)

Introduce the concept of FOIL using the following example. Students can use this technique to ensure all factors are multiplied.

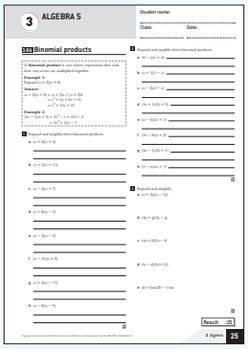


Digital resources

ProductLink

- Binomial products (Drag-and-drop)

Homework 3:06



WORKED EXAMPLES

1 $(a+2)(b+4) = a(b+4) + 2(b+4)$
$= ab + 4a + 2b + 8$

2 $(a-2)(a+7) = a(a+7) - 2(a+7)$
$= a^2 + 7a - 2a - 14$
$= a^2 + 5a - 14$

3 $(x+2y)(2x+y) = x(2x+y) + 2y(2x+y)$
$= 2x^2 + xy + 4xy + 2y^2$
$= 2x^2 + 5xy + 2y^2$

4 $(1-x)(x-3) = 1(x-3) - x(x-3)$
$= x - 3 - x^2 + 3x$
$= 4x - x^2 - 3$

You should notice that each term in the first binomial is multiplied by each term in the second; i.e.

$(x+5)(2x-3)$ with $2x^2$, -15, $10x$, $-3x$

$= 2x^2 + 10x - 3x - 15$
$= 2x^2 + 7x - 15$

$(a+b)(c+d) = a(c+d) + b(c+d)$
$= ac + ad + bc + bd$

Exercise 3:06

1 Expand the following binomial products.

a $(a+2)(b+3)$	b $(x+1)(y+4)$	c $(m+7)(n+5)$	d $(a+3)(x+2)$
e $(p+5)(q+4)$	f $(2x+1)(y+3)$	g $(a+6)(3p+2)$	h $(4x+1)(2y+3)$
i $(3a+1)(2b-7)$	j $(7x+5)(2p+1)$	k $(5p+3)(x-4)$	l $(2x+y)(a+2b)$

2 Expand the following and collect the like terms.

a $(a+2)(a+3)$	b $(x+1)(x+5)$	c $(n+3)(n+4)$	d $(p+2)(p+5)$
e $(m+1)(m-3)$	f $(y+7)(y-2)$	g $(x+1)(x-6)$	h $(t+2)(t-4)$
i $(x-2)(x-4)$	j $(n-7)(n-1)$	k $(a-6)(a-3)$	l $(x-10)(x-9)$
m $(y-11)(y+7)$	n $(a-2)(a+1)$	o $(x-8)(x-8)$	p $(m-9)(m-2)$
q $(a-3)(a+3)$	r $(x-7)(x+3)$	s $(y+12)(y+5)$	t $(a-8)(a+8)$
u $(q+5)(q+5)$	v $(x-1)(x-9)$	w $(t+3)(t+10)$	x $(k-8)(k+11)$

3 Find these products and simplify.

a $(a+3)(2a+1)$	b $(2x+1)(x+2)$	c $(3m+2)(m+5)$
d $(y+3)(4y+1)$	e $(2x+1)(2x+3)$	f $(3n+2)(2n+1)$
g $(2x+3)(4x+3)$	h $(5t+2)(2t+3)$	i $(2x-2)(5x-1)$
j $(8p+1)(3p-2)$	k $(5m-2)(2m-5)$	l $(3q+1)(7q-2)$
m $(3x+2)(6x-2)$	n $(2n+3)(2n-3)$	o $(8y-1)(8y+1)$
p $(3k-2)(5k-3)$	q $(7p-1)(7p-1)$	r $(3x-1)(5x-3)$
s $(5x+4)(5x+4)$	t $(9y-4)(3y+2)$	u $(5p+2)(p-7)$
v $(10q-1)(q-10)$	w $(4a+3)(3a+4)$	x $(7p+5)(7p-5)$

Answers

Exercise 3:06

1 a $ab + 3a + 2b + 6$
b $xy + 4x + y + 4$
c $mn + 5m + 7n + 35$
d $ax + 2a + 3x + 6$
e $pq + 4p + 5q + 20$
f $2xy + 6x + y + 3$
g $3ap + 2a + 18p + 12$
h $8xy + 12x + 2y + 3$
i $6ab - 21a + 2b - 7$
j $14px + 7x + 10p + 5$
k $5px - 20p + 3x - 12$
l $2ax + 4bx + ay + 2by$

2 a $a^2 + 5a + 6$
b $x^2 + 6x + 5$
c $n^2 + 7n + 12$
d $p^2 + 7p + 10$
e $m^2 - 2m - 3$
f $y^2 + 5y - 14$
g $x^2 - 5x - 6$
h $t^2 - 2t - 8$
i $x^2 - 6x + 8$
j $n^2 - 8n + 7$
k $a^2 - 9a + 18$
l $x^2 - 19x + 90$
m $y^2 - 4y - 77$
n $a^2 - a - 2$
o $x^2 - 16x + 64$
p $m^2 - 11m + 18$
q $a^2 - 9$
r $x^2 - 4x - 21$
s $y^2 + 17y + 60$
t $a^2 - 64$
u $q^2 + 10q + 25$
v $x^2 - 10x + 9$
w $t^2 + 13t + 30$
x $k^2 + 3k - 88$

3 a $2a^2 + 7a + 3$
b $2x^2 + 5x + 2$
c $3m^2 + 17m + 10$
d $4y^2 + 13y + 3$
e $4x^2 + 8x + 3$
f $6n^2 + 7n + 2$
g $8x^2 + 18x + 9$
h $10t^2 + 19t + 6$
i $10x^2 - 12x + 2$
j $24p^2 - 13p - 2$
k $10m^2 - 29m + 10$
l $21q^2 + q - 2$
m $18x^2 + 6x - 4$
n $4n^2 - 9$
o $64y^2 - 1$
p $15k^2 - 19k + 6$
q $49p^2 - 14p + 1$
r $15x^2 - 14x + 3$
s $25x^2 + 40x + 16$
t $27y^2 + 6y - 8$
u $5p^2 - 33p - 14$
v $10q^2 - 101q + 10$
w $12a^2 + 25a + 12$
x $49p^2 - 25$

Teacher's notes

4 Expand and simplify:

a $(3 + x)(4 + x)$	**b** $(5 - a)(2 - a)$	**c** $(7 + m)(1 - m)$
d $(3 - n)(3 + n)$	**e** $(4 + y)(y + 5)$	**f** $(x - 7)(5 - x)$
g $(9 + k)(k + 10)$	**h** $(2a + 1)(3 + a)$	**i** $(3n + 1)(7 - 2n)$
j $(x + y)(x + 2y)$	**k** $(2n + m)(n + 2m)$	**l** $(a - b)(2a + 3b)$
m $(2p - q)(2p + q)$	**n** $(3x + y)(2x - 5y)$	**o** $(3a + 2b)(2a + 3b)$
p $(9w - 5x)(9w - 5x)$		

3:07 Special products

3:07A Perfect squares

PREP QUIZ 3:07A

Simplify:

1 4^2 **2** 7^2 **3** $(-2)^2$ **4** $(-10)^2$ **5** $(3x)^2$

Complete:

6 $(x + 7)(x + 7) = x^2 + 14x + \ldots$

7 $(a - 3)(a - 3) = a^2 - 6a + \ldots$

8 $(2m - 1)(2m - 1) = \ldots m^2 - 4m + 1$

9 $(n + 5)(n + 5) = n^2 + \ldots n + 25$

10 $(x - 3)(x - 3) = x^2 - \ldots x + 9$

When a binomial is multiplied by itself, we call this product a perfect square. If a perfect square is expanded, we get:

$$\begin{aligned}(x + y)^2 &= (x + y)(x + y)\\ &= x(x + y) + y(x + y)\\ &= x^2 + xy + yx + y^2\\ &= x^2 + 2xy + y^2\end{aligned}$$

Similarly: $(x - y)^2 = x^2 - 2xy + y^2$

The square of a binomial is equal to the square of the first term, plus twice the product of the two terms, plus the square of the second term.

$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$

Answers

Exercise 3:06

4

a	$12 + 7x + x^2$	**b**	$10 - 7a + a^2$
c	$7 - 6m - m^2$	**d**	$9 - n^2$
e	$y^2 + 9y + 20$	**f**	$12x - x^2 - 35$
g	$k^2 + 19k + 90$	**h**	$2a^2 + 7a + 3$
i	$19n - 6n^2 + 7$	**j**	$x^2 + 3xy + 2y^2$
k	$2n^2 + 5mn + 2m^2$	**l**	$2a^2 + ab - 3b^2$
m	$4p^2 - q^2$	**n**	$6x^2 - 13xy - 5y^2$
o	$6a^2 + 13ab + 6b^2$	**p**	$81w^2 - 90wx + 25x^2$

3:07 Content statements

Expand binomial products using a variety of strategies (ACMNA233) [Stage 5.3^{S}]

- recognise and apply the special products,
  $(a + b)^2 = a^2 + 2ab + b^2$
  $(a - b)^2 = a^2 - 2ab + b^2$
- use algebraic methods to expand a variety of binomial products, including the special products, eg $(2y + 1)^2$, $(3a - 1)(3a + 1)$
- simplify a variety of expressions involving binomial products, eg
  $(3x + 1)(2 - x) + 2x + 4$,
  $(x - y)^2 - (x + y)^2$

Answers

PREP QUIZ 3:07A

1 16	**2** 49	**3** 4	**4** 100
5 $9x^2$	**6** 49	**7** 9	**8** 4
9 10	**10** 6		

Lesson starter

Using FOIL for perfect squares

Using the FOIL method (see the Teaching strategy on page 65), have students try to expand a number of perfect squares from Exercise 3:07A Question 1.

Teacher's notes

Digital resources

ProductLink

- Special products [Drag-and-drop]

Teaching strategies

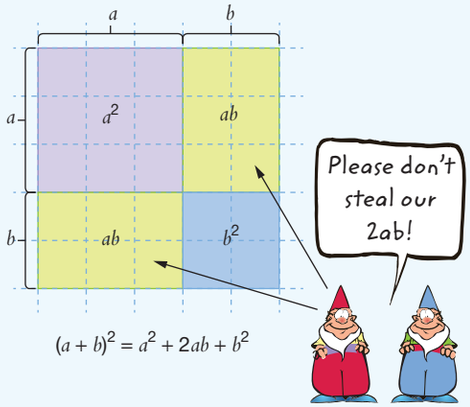
Perfect square rule

Many students will continue to expand perfect squares using FOIL. Students need to practise and be encouraged to use the perfect square rule.

Misconception: $(3x + 4)^2 = 3x^2 + 16$

Point out to students the common error when expanding perfect squares. Explain to the students why this example is incorrect.

$(3x + 4)^2$
$= 3x^2 + 4^2$
$= 9x^2 + 16$



Therefore:

$(3x + 4)^2$
$= (3x + 4)(3x + 4)$
$= 9x^2 + 12x + 12x + 16$
$= 9x^2 + 24x + 16$

Investigation 3:07

- Remind students that $6^2 = 36$.

  A common error is that students calculate $6^2 = 12$.

- Remind students that $(x + y)^2 = x^2 + 2xy + y^2$.

  A common error is that students expand $(x + y)^2 = x^2 + y^2$.

Answers

Exercise 3:07A

1	a	4	b	36	c	9	d	100
	e	$2x$	f	$14y$	g	$4n$	h	$10p$
	i	q^2	j	x^2	k	3	l	9
	m	7	n	11	o	$4x^2$	p	$25n^2$
	q	$42m$	r	$40x$	s	$4a$	t	$126y$

WORKED EXAMPLES

1 $(a + 3)^2 = a^2 + 2[3a] + 3^2$
$= a^2 + 6a + 9$

first term squared; product of the two terms; second term squared

2 $(m - 5)^2 = m^2 - 2[5m] + 5^2$
$= m^2 - 10m + 25$

first term squared; twice the product of the two terms; second term squared

3 $(3y - 7)^2 = [3y]^2 - 2[21y] + [-7]^2$
$= 9y^2 - 42y + 49$

INVESTIGATION 3:07 THE SQUARE OF A BINOMIAL

The Prep quiz on the previous page suggests that there might be a pattern formed when a binomial is squared.

Copy and complete this table.

x	y	x^2	y^2	xy	$(x + y)^2$	$x^2 + 2xy + y^2$	$(x - y)^2$	$x^2 - 2xy + y^2$
5	3							
6	1							
10	4							

What are your findings?

Exercise 3:07A

1 Find the missing term in each example to make the statement true.

a	$(x + 2)^2 = x^2 + 4x + \ldots$	b	$(a + 6)^2 = a^2 + 12a + \ldots$
c	$(y - 3)^2 = y^2 - 6y + \ldots$	d	$(m - 10)^2 = m^2 - 20m + \ldots$
e	$(x + 1)^2 = x^2 + \ldots + 1$	f	$(y + 7)^2 = y^2 + \ldots + 49$
g	$(n - 2)^2 = n^2 - \ldots + 4$	h	$(p - 5)^2 = p^2 - \ldots + 25$
i	$(q + 8)^2 = \ldots + 16q + 64$	j	$(x - 4)^2 = \ldots - 8x + 16$
k	$(x + \ldots)^2 = x^2 + 6x + 9$	l	$(a + \ldots)^2 = a^2 + 18a + 81$
m	$(y - \ldots)^2 = y^2 - 14x + 49$	n	$(m - \ldots)^2 = m^2 - 22m + 121$
o	$(2x + 3)^2 = \ldots + 12x + 9$	p	$(5n + 1)^2 = \ldots + 10n + 1$
q	$(3m + 7)^2 = 9m^2 + \ldots + 49$	r	$(4x + 5)^2 = 16x^2 + \ldots + 25$
s	$(2a - 1)^2 = 4a^2 - \ldots + 1$	t	$(9y - 7)^2 = 81y^2 - \ldots + 49$

Answers

INVESTIGATION 3:07

x	y	x^2	y^2	xy	$(x+y)^2$	$x^2+2xy+y^2$	$(x-y)^2$	$x^2-2xy+y^2$
5	3	25	9	15	64	64	4	4
6	1	36	1	6	49	49	25	25
10	4	100	16	40	196	196	36	36

2 Expand these perfect squares and simplify.

a $(x+3)^2$	b $(x+5)^2$	c $(x+1)^2$
d $(x-6)^2$	e $(m-1)^2$	f $(n-5)^2$
g $(x+2)^2$	h $(n-8)^2$	i $(m+11)^2$
j $(a+12)^2$	k $(x+10)^2$	l $(p-9)^2$
m $(x+y)^2$	n $(a+m)^2$	o $(x+t)^2$
p $(a-b)^2$	q $(k-m)^2$	r $(p-q)^2$

3 Expand and simplify:

a $(2x+3)^2$	b $(2x+1)^2$	c $(3x+5)^2$
d $(4a+1)^2$	e $(3a+7)^2$	f $(7t+2)^2$
g $(2x-1)^2$	h $(3a-2)^2$	i $(5m-4)^2$
j $(4t-7)^2$	k $(6q-1)^2$	l $(9n+4)^2$
m $(2x+y)^2$	n $(a+3b)^2$	o $(3t-2x)^2$

3:07B Difference of two squares

PREP QUIZ 3:07B

Evaluate:

1 7^2-3^2	2 $(7+3)(7-3)$
3 4^2-2^2	4 $(4+2)(4-2)$
5 5^2-1^2	6 $(5-1)(5+1)$
7 6^2-3^2	8 $(6-3)(6+3)$
9 10^2-9^2	10 $(10+9)(10-9)$

If the sum of two terms is multiplied by their difference, another special type of product is formed. If $(x+y)$ is multiplied by $(x-y)$ we get:

$$\begin{aligned}(x+y)(x-y) &= x(x-y)+y(x-y)\\ &= x^2-xy+yx-y^2\\ &= x^2-y^2\end{aligned}$$

The sum of two terms multiplied by their difference is equal to the square of the first term minus the square of the second term.

$$(x+y)(x-y)=x^2-y^2$$

Teacher's notes

Answers

Exercise 3:07A

2 a x^2+6x+9	b $x^2+10x+25$	
c x^2+2x+1	d $x^2-12x+36$	
e m^2-2m+1	f $n^2-10n+25$	
g x^2+4x+4	h $n^2-16n+64$	
i $m^2+22m+121$	j $a^2+24a+144$	
k $x^2+20x+100$	l $p^2-18p+81$	
m $x^2+2xy+y^2$	n $a^2+2am+m^2$	
o $x^2+2xt+t^2$	p $a^2-2ab+b^2$	
q $k^2-2km+m^2$	r $p^2-2pq+q^2$	
3 a $4x^2+12x+9$	b $4x^2+4x+1$	
c $9x^2+30x+25$	d $16a^2+8a+1$	
e $9a^2+42a+49$	f $49t^2+28t+4$	
g $4x^2-4x+1$	h $9a^2-12a+4$	
i $25m^2-40m+16$	j $16t^2-56t+49$	
k $36q^2-12q+1$	l $81n^2+72n+16$	
m $4x^2+4xy+y^2$	n $a^2+6ab+9b^2$	
o $9t^2-12xt+4x^2$		

Answers

PREP QUIZ 3:07B

1 40	2 40	3 12	4 12
5 24	6 24	7 27	8 27
9 19	10 19		

Teaching strategies

Difference of two squares (DOTS) rule

Many students will continue expanding DOTS using FOIL. Students need to practise and be encouraged to use the DOTS rule.

Common errors

Point out to students the common error when expanding DOTS. Explain to the students why this example is incorrect.

$(2x-5)(2x+5)$

$=2x^2-25$

Interesting fact

A binomial contains two terms (e.g. $2r+by$). A trinomial contains three terms.

Any expression containing two or more terms may also be called by the general name polynomial (*poly* means many).

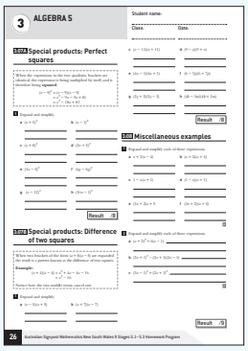
Answers

Exercise 3:07B

1 a $x^2 - 16$ b $a^2 - 1$
c $m^2 - 4$ d $n^2 - 49$
e $p^2 - 25$ f $q^2 - 36$
g $x^2 - 9$ h $y^2 - 81$
i $100 - x^2$ j $25 - a^2$
k $64 - x^2$ l $121 - m^2$
m $x^2 - t^2$ n $a^2 - b^2$
o $m^2 - n^2$ p $p^2 - q^2$

2 a $4a^2 - 1$ b $9x^2 - 4$
c $25m^2 - 9$ d $81q^2 - 4$
e $16t^2 - 9$ f $49x^2 - 1$
g $64n^2 - 25$ h $100x^2 - 9$
i $4x^2 - y^2$ j $16a^2 - 9b^2$
k $25p^2 - 4q^2$ l $9m^2 - n^2$
m $4m^2 - 25n^2$ n $4p^2 - 9q^2$
o $x^2 - 25y^2$ p $144x^2 - 25y^2$

Homework 3:07


3 ALGEBRA 5

Student name:

Class Date

3.07 Special products: Perfect squares

Result /8

3.08 Miscellaneous examples

Result /3

3.07 Special products: Difference of two squares

Result /3

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WORKED EXAMPLES

1 $(x + 3)(x - 3) = x^2 - 3^2$
first term squared; second term squared
$= x^2 - 9$

2 $(2a - 3b)(2a + 3b) = (2a)^2 - (3b)^2$
first term squared; second term squared
$= 4a^2 - 9b^2$

3 $(p - 7)(p + 7) = p^2 - 7^2$
$= p^2 - 49$

4 $(5x + y)(5x - y) = (5x)^2 - y^2$
$= 25x^2 - y^2$

Exercise 3:07B

1 Expand these products and simplify.

a $(x + 4)(x - 4)$ b $(a + 1)(a - 1)$ c $(m + 2)(m - 2)$ d $(n + 7)(n - 7)$
e $(p - 5)(p + 5)$ f $(q - 6)(q + 6)$ g $(x - 3)(x + 3)$ h $(y - 9)(y + 9)$
i $(10 + x)(10 - x)$ j $(5 + a)(5 - a)$ k $(8 - x)(8 + x)$ l $(11 - m)(11 + m)$
m $(x + t)(x - t)$ n $(a - b)(a + b)$ o $(m + n)(m - n)$ p $(p - q)(p + q)$

2 Express as the difference of two squares.

a $(2a + 1)(2a - 1)$ b $(3x + 2)(3x - 2)$ c $(5m + 3)(5m - 3)$
d $(9q + 2)(9q - 2)$ e $(4t - 3)(4t + 3)$ f $(7x - 1)(7x + 1)$
g $(8n - 5)(8n + 5)$ h $(10x - 3)(10x + 3)$ i $(2x + y)(2x - y)$
j $(4a + 3b)(4a - 3b)$ k $(5p + 2q)(5p - 2q)$ l $(3m - n)(3m + n)$
m $(2m - 5n)(2m + 5n)$ n $(2p - 3q)(2p + 3q)$ o $(x - 5y)(x + 5y)$
p $(12x - 5y)(12x + 5y)$

3:08 Miscellaneous examples

- It is important that you are able to expand and simplify algebraic expressions readily and accurately, if you are to use algebra in later problem-solving exercises.
- Work through the following miscellaneous questions after examining the following two examples.

WORKED EXAMPLES

1 $(x + 3)^2 - (x - 1)(x + 2) = [x^2 + 6x + 9] - [x^2 + x - 2]$
$= x^2 + 6x + 9 - x^2 - x + 2$
$= 5x + 11$

2 $(3x + 5)(x - 1) + (x + 2)^2 - (2x + 1)(2x - 1) = [3x^2 + 2x - 5] + [x^2 + 4x + 4] - [4x^2 - 1]$
$= 3x^2 + 2x - 5 + x^2 + 4x + 4 - 4x^2 + 1$
$= 6x$

3:08 Content statements

Expand binomial products using a variety of strategies (ACMNA233) [Stage 5.3§]

- use algebraic methods to expand a variety of binomial products, including the special products, eg $(2y + 1)^2$, $(3a - 1)(3a + 1)$
- simplify a variety of expressions involving binomial products, eg $(3x + 1)(2 - x) + 2x + 4$, $(x - y)^2 - (x + y)^2$

Exercise 3:08

Expand and simplify, where possible, each of the following expressions.

1 a $5x + 3(x - 7)$ b $(x + 2)(x - 1)$ c $(2x + 1)(x - 1)$
d $5(x + 2) - x(x + 1)$ e $(3x - 1)^2$ f $(x + 5)(x - 5)$
g $(2x - 7)(3x - 1)$ h $(5x - 1)(5x + 1)$ i $4x + 7 + x(x + 2)$
j $9x - (x + 5) + 5$ k $(x + 10)(x - 3)$ l $(9 - y)(9 + y)$
m $3x(x - 5) - 2x^2$ n $3(x + 2)(x + 1)$ o $(x + y)^2$
p $(x + 2y)(2x + y)$ q $5x - 2(x + y) + 2y$ r $(a + 2b)(a - 2b)$
s $a(x + 2) - x(a + 2)$ t $(3a + 7)(5a - 3)$ u $(2m - 5n)^2$
v $(1 - 5y)(1 + 5y)$ w $3x - 7(x - 3)$ x $(9x - 8y)(9x + 8y)$

2 a $(x + 1)^2 + 5(x + 2)$ b $(a - 3)^2 - 3(a + 1)$
c $(x + 2)(x + 3) - 7(x - 2)$ d $8(x + 2) + (x - 7)(x + 1)$
e $(x + 3)^2 + (x + 1)(x + 2)$ f $(a + 5)(a + 3) - (a + 4)^2$
g $(m + 6)^2 - (m - 1)(m + 1)$ h $(y + 7)(y - 7) - (y + 7)^2$
i $(x + 2)^2 + (x + 1)^2$ j $(a + 3)^2 - (a + 2)^2$
k $(x + 1)(x + 2) + (x + 2)(x + 3)$ l $(a + 1)(a - 2) + (a + 2)(a - 1)$
m $(x + 3)(x - 1) - (x + 2)(x - 5)$ n $(y + 7)(y - 2) - (y + 1)(y + 3)$
o $(2x + 1)^2 - 5(x + 3)$ p $2x(x + 5) + (x + 7)^2$
q $(5x + 1)(x - 3) + (2x + 1)^2$ r $(2x + 1)(3x + 1) - (2x - 1)(3x - 1)$
s $(p + 3)(p - 3) - (q + 3)(q - 3)$ t $(x + y)^2 - (x - y)(x + y)$
u $(a + b)(a + 2b) + (a + b)^2$ v $(m - n)^2 + (m + n)^2$
w $3(x + 1)^2 + 5(x + 1)$ x $2(x - 1)(x + 1) + 3(x + 1)^2$
y $(2x + 3y)^2 - (2x - 3y)(2x + 3y)$ z $(3a + 2b)(2a + 3b) - 6(a + b)^2$

3 a $(x + 1)^2 + (x + 2)^2 + (x + 3)^2$
b $(x + 1)(x + 2) + (x + 2)(x + 3) + (x + 3)(x + 4)$
c $(a - 1)(a + 1) + (a + 1)^2 + (a - 1)^2$
d $(x + 2)^2 + (x + 3)^2 - (x + 2)(x + 3)$
e $(3a + 2b)(2a + 3b) + (3a - 2b)(3a + 2b) + (2a + 3b)(2a - 3b)$
f $(4x + 1)(3x - 1) + (x + 2)^2 - (x - 3)(x + 3)$
g $5(m - 5)^2 - 8(m - 4)^2 + 3(m - 3)^2$
h $(3x + 2y)(3x - 2y) - (2x + y)(2x - y) - (x + 1)(x - 1)$
i $(x + 3y)^2 - (2x + 2y)^2 + (3x + y)^2$
j $2(x - y)(x + y) - (x + y)^2 - (x - y)^2$

CHALLENGE 3:08 PATTERNS IN PRODUCTS

The examples below involve the sum of a series of products. Can you see the patterns involved and, hence, find the simplest expression for each sum?

1 $(x + 1)^2 + (x + 2)^2 \ldots + (x + 9)^2 + (x + 10)^2$
2 $(x + 1)(x + 2) + (x + 2)(x + 3) + \ldots + (x + 9)(x + 10)$
3 $(a - 5)^2 + (a - 4)^2 + \ldots + a^2 + \ldots + (a + 4)^2 + (a + 5)^2$
4 $(5m - n)(5m + n) + (4m - 2n)(4m + 2n) + (3m - 3n)(3m + 3n) + (2m - 4n)(2m + 4n) + (m - 5n)(m + 5n)$

Answers

CHALLENGE 3:08

1 $10x^2 + 110x + 385$
2 $9x^2 + 99x + 330$
3 $11a^2 + 110$
4 $55m^2 - 55n^2$

Lesson starter

Rule revision

As a lesson starter have students revise the rules for simplifying expressions.

- $a(b + c)$ Use distributive law
- $(a + b)(c + d)$ Use FOIL
- $(a + b)^2$ Use perfect square rule
- $(a - b)^2$ Use perfect square rule
- $(a - b)(a + b)$ Use DOTS rule

Teaching strategies

Encourage students to use all of the rules developed to expand a variety of binomial expressions.

- $a(b + c) = ab + ac$
- $(a + b)(c + d) = ac + ad + bc + bd$
- $(a - b)(c - d) = ac - ad - bc + bd$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a - b)(a + b) = a^2 - b^2$

Answers

Exercise 3:08

1 a $8x - 21$ b $x^2 + x - 2$
c $2x^2 - x - 1$ d $4x - x^2 + 10$
e $9x^2 - 6x + 1$ f $x^2 - 25$
g $6x^2 - 23x + 7$ h $25x^2 - 1$
i $x^2 + 6x + 7$ j $8x$
k $x^2 + 7x - 30$ l $81 - y^2$
m $x^2 - 15x$ n $3x^2 + 9x + 6$
o $x^2 + 2xy + y^2$ p $2x^2 + 5xy + 2y^2$
q $3x$ r $a^2 - 4b^2$
s $2a - 2x$ t $15a^2 + 26a - 21$
u $4m^2 - 20mn + 25n^2$ v $1 - 25y^2$
w $21 - 4x$ x $81x^2 - 64y^2$

2 a $x^2 + 7x + 11$ b $a^2 - 9a + 6$
c $x^2 - 2x + 20$ d $x^2 + 2x + 9$
e $2x^2 + 9x + 11$ f -1
g $12m + 37$ h $-14y - 98$
i $2x^2 + 6x + 5$ j $2a + 5$
k $2x^2 + 8x + 8$ l $2a^2 - 4$
m $5x + 7$ n $y - 17$
o $4x^2 - x - 14$ p $3x^2 + 24x + 49$
q $9x^2 - 10x - 2$ r $10x$
s $p^2 - q^2$ t $2xy + 2y^2$
u $2a^2 + 5ab + 3b^2$ v $2m^2 + 2n^2$
w $3x^2 + 11x + 8$ x $5x^2 + 6x + 1$
y $12xy + 18y^2$ z ab

3 a $3x^2 + 12x + 14$ b $3x^2 + 15x + 20$
c $3a^2 + 1$ d $5x + 7$
e $19a^2 + 13ab - 7b^2$ f $12x^2 + 3x + 12$
g $-4m + 24$ h $4x^2 - 3y^2 + 1$
i $6x^2 + 4xy + 6y^2$ j $-4y^2$

Extension

Binomial expansions and Pascal's triangle

Have students investigate and write a report on how they can use Pascal's triangle to expand the following expressions.

1 $(1 + 3x)^2$
2 $(2 + x)^3$
3 $(1 - x)^3$
4 $(1 - 5x)^5$
5 $(x + 6)^3$
6 $(a - b)^7$
7 $(1 + \frac{3}{a})^4$

[Understanding, Communicating]

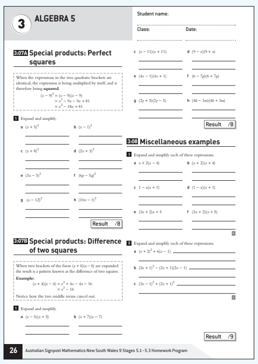
Learning across the curriculum

Literacy

Have students explain how to expand expressions using the rules for FOIL, perfect squares and DOTS, showing the steps in detail.

[Understanding, Fluency]

Homework 3:08



Answers

INVESTIGATION 3:08

Perfect squares

1 a 10 201 b 42 025 c 1 008 016
d 5184 e 9604 f 39 601
g 990 025 h 4489

Difference of two squares

1 a 396 b 840 c 1425
d 12 920
2 a $\sqrt{56}$ b $\sqrt{540}$ c $\sqrt{445}$
d $\sqrt{960}$

INVESTIGATION 3:08 USING SPECIAL PRODUCTS IN ARITHMETIC

Perfect squares

Example

Using $(a \pm b)^2 = a^2 \pm 2ab + b^2$, evaluate $(103)^2$.

Solution

Writing 103 as (100 + 3)

Then $103^2 = (100 + 3)^2$
$= 100^2 + 2 \times 100 \times 3 + 3^2$
$= 10\,000 + 600 + 9$
$= 10\,609$

Similarly, the square of a number like 98 could be found by writing 98 as (100 – 2).

Exercise

1 Following the example above, evaluate:
a 101^2 b 205^2 c 1004^2 d 72^2
e 98^2 f 199^2 g 995^2 h 67^2

Difference of two squares

Example

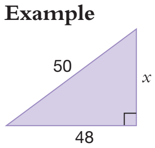
Using $(a - b)(a + b) = a^2 - b^2$, evaluate $100^2 - 97^2$.

Solution

$100^2 - 97^2 = (100 - 97)(100 + 97)$
$= 3 \times 197$
$= 591$

This method can be useful when finding a shorter side of a right-angled triangle.

Example



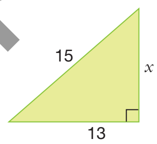
Solution

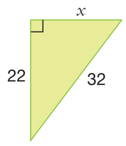
$x^2 = 50^2 - 48^2$
$= (50 - 48)(50 + 48)$
$= 2 \times 98$
$= 196$
$\therefore x = \sqrt{196}$
$= 14$

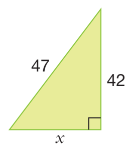
Exercise

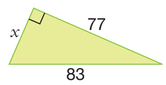
1 Evaluate:
a $100^2 - 98^2$ b $73^2 - 67^2$ c $145^2 - 140^2$ d $651^2 - 641^2$

2 Use the method above to find the value of x for each triangle. (Leave your answer in surd form.)

a


b


c


d


Teacher's notes

MATHS TERMS 3

algebra
- a branch of mathematics where numbers are represented by symbols

algebraic expression
- a group of terms and numbers that are joined by addition or subtraction signs

binomial
- an algebraic expression consisting of two terms,
  e.g. $2x + 4, 3x - 2y$

brackets
- the name given to these grouping symbols: [ ]

cancel
- to simplify a fraction by dividing the numerator and denominator by any common factor,
  e.g. $\frac{\overset{7}{\cancel{21} \div 3}}{\underset{10}{\cancel{30} \div 3}}$ so $\frac{21}{30} = \frac{7}{10}$

collect like terms
- to simplify an algebraic expression containing many terms by addition and/or subtraction,
  e.g. $5x + 3 + 7x - 4$
  $= 12x - 1$

difference of two squares
- the result of multiplying two binomials which are the sum and difference of the same terms,
  e.g. $(a + 3)(a - 3) = a^2 - 3^2$
  $= a^2 - 9$

denominator
- the bottom number of a fraction

expand
- to remove grouping symbols by multiplying each term inside the grouping symbols by the term outside

factorise
- to write an expression as a product
- the reverse of expanding

like terms
- terms that have identical pronumeral parts,
  e.g. $7x$ and $10x$, $5a^2b$ and $-3a^2b$

numerator
- the top number of a fraction

parentheses
- the name given to these grouping symbols: ( )

perfect square
- when a binomial is multiplied by itself,
  e.g. $(x + 5)^2$ or $(2a - 3b)^2$

pronumeral
- a symbol, usually a letter, used to represent a number

substitution
- the replacing of a pronumeral with a numeral in an expression,
  e.g. to substitute 3 for a in the expression $4a - 2$ would give:
  $4(3) - 2 = 10$

A machine counts coins by weight. What is the value of a pile of $M coins that weighs W grams if each coin weighs w grams?

Learning across the curriculum

Literacy

In groups, have students create a crossword using the words in Maths terms 3. Free crossword puzzle generators can be found on the internet. Once completed, each group must solve another group's puzzle.

[Understanding]

Technology

Prime factorisation

Search the internet for 'prime factorisation tool'. Try to find your unique 'birthday' set of prime factors using technology. For example, the birth date 7/5/1995 would be the number 751995.

Extension

Factorisation using grouping 'two and two'

Research how you can factorise the following expressions.

	Answers:
1 $4y + cy + 2c + 8$	**1** $(y + 2)(c + 4)$
2 $a^2 - 3a - 6 + 2a$	**2** $(a - 3)(a + 2)$
3 $5a - ab - 2b + 10$	**3** $(a + 2)(5 - b)$
4 $3z + 2y + 6 + yz$	**4** $(z + 2)(3 + y)$

Learning across the curriculum

Critical and creative thinking

Discuss and explain when factorising $36a^2 + 18a$, why $6(6a^2 + 3a)$ is not the best answer.

[Understanding, Reasoning]

Teacher's notes

Digital resources

ProductLink
- Maths terms 3 (Drag-and-drop)

Answers

DIAGNOSTIC TEST 3

1 a $2a + 3b$ b $2p^2 + 2p$
c $2ab$ d $4a - x - 2$

2 a $56m$ b $30ab$
c $10y^2$ d $-8ny$

3 a $3a$ b $5y$
c $\frac{3c}{2b}$ d $\frac{-1}{3y}$

4 a x b $-\frac{x}{6}$
c $\frac{7a}{15}$ d $\frac{9m}{8}$

5 a $\frac{n}{4}$ b $\frac{10}{ab}$
c $\frac{1}{2}$ d $\frac{6}{5}$

6 a $6m$ b 2
c 12 d $\frac{5a}{2}$

7 a $9x + 63$ b $30a - 12$
c $p^2 + 3p$ d $15a - 6a^2$

8 a $-3x - 6$ b $-2m + 16$
c $-15x - 20$ d $-7 + 2m$

9 a $-x$ b $10n - 7$
c $-a^2 + 6ab$

10 a $\frac{7x + 26}{10}$ b $\frac{12a + 1}{12}$
c $\frac{5n + 11}{12}$

11 a $5(m + 2)$ b $x(x - 3)$
c $3a(2b + 5)$ d $-4(2y + 3)$

12 a $x^2 + 7x + 12$ b $2a^2 - 7a + 3$
c $6 - y - y^2$ d $2x^2 - 5xy - 3y^2$

13 a $x^2 + 4x + 4$ b $a^2 - 14a + 49$
c $4y^2 + 20y + 25$ d $m^2 - 2mn + n^2$

14 a $x^2 - 9$ b $y^2 - 49$
c $4a^2 - 25$ d $x^2 - y^2$

Assessment

Diagnostic test 3

Split the test in half. Do the first two columns of Questions 2 to 11 as a test. Have each student mark their own paper, giving themselves a score. Students complete their own peer assessment on each section of the algebra unit by writing reflective comments on which sections they did well in and which sections they need to revise. Design a summary sheet with examples and comments on how to complete the problems before completing the second half of the test. Again, have students mark their own paper.

[Understanding, Communicating]

DIAGNOSTIC TEST 3 ALGEBRA

Each part of this test has similar items that test a certain type of example.
Errors made will indicate areas of weakness.
Each weakness should be treated by going back to the section listed.

1 Simplify:		3:01
a $5a + 2b - 3a + b$	b $5p^2 + 2p - 3p^2$	
c $6ab - 4ba$	d $6a - 2x + 5 + x - 2a - 7$	
2 Simplify:		3:01
a $8 \times 7m$	b $5a \times 6b$	c $10y \times y$ d $-4n \times 2y$
3 Simplify:		3:01
a $6a \div 2$	b $15xy \div 3x$	c $12ac \div 8ab$ d $-6x \div 18xy$
4 Simplify:		3:02A
a $\frac{3x}{5} + \frac{2x}{5}$	b $\frac{x}{3} - \frac{x}{2}$	c $\frac{4a}{5} - \frac{a}{3}$ d $\frac{5m}{8} + \frac{m}{2}$
5 Simplify:		3:02B
a $\frac{3}{4} \times \frac{n}{3}$	b $\frac{2}{a} \times \frac{5}{b}$	c $\frac{5}{x} \times \frac{x}{10}$ d $\frac{3b}{2} \times \frac{4}{5b}$
6 Simplify completely:		3:04B
a $\frac{3m}{2} \div \frac{1}{4}$	b $\frac{x}{3} \div \frac{x}{6}$	c $\frac{8a}{3b} \div \frac{2a}{9b}$ d $\frac{ab}{2} \div \frac{b}{5}$
7 Expand:		3:03
a $9(x + 7)$	b $6(5a - 2)$	c $p(p + 3)$ d $3a(5 - 2a)$
8 Expand:		3:03
a $-3(x + 2)$	b $-2(m - 8)$	c $-5(3x + 4)$ d $-(7 - 2m)$
9 Expand and simplify:		3:03
a $x(x - 1) - x^2$	b $7n - 4 + 3(n - 1)$	c $2a(a + b) - a(3a - 4b)$
10 Simplify:		3:04
a $\frac{x + 4}{2} + \frac{x + 3}{5}$	b $\frac{2a - 5}{4} + \frac{3a + 8}{6}$	c $\frac{3n + 1}{4} - \frac{n - 2}{3}$
11 Factorise completely:		3:05
a $5m + 10$	b $x^2 - 3x$	c $6ab + 15a$ d $-8y - 12$
12 Expand and simplify:		3:06
a $(x + 3)(x + 4)$	b $(a - 3)(2a - 1)$	c $(2 - y)(3 + y)$ d $(2x + y)(x - 3y)$
13 Expand and simplify:		3:07A
a $(x + 2)^2$	b $(a - 7)^2$	c $(2y + 5)^2$ d $(m - n)^2$
14 Expand and simplify:		3:07B
a $(x + 3)(x - 3)$	b $(y - 7)(y + 7)$	c $(2a + 5)(2a - 5)$ d $(x + y)(x - y)$

Teacher's notes

ASSIGNMENT 3A Chapter review

1 Simplify the following.

a $6a + a$ **b** $6x \times 3x$
c $a - 5a$ **d** $x^2 + x^2$
e $18x \div 3x$ **f** $12y \div 8$
g $2x + 3y$ **h** $3ab \times 2b$
i $12a^2b \div 6a$ **j** $5ab + 7ab$
k $6a^2 - a$ **l** $4x - 3y - 5x$
m $12 + 6x + 7 - x$ **n** $6x + 2x \times 3$
o $x^2 - 3x + 2x + 3x^2$ **p** $12x - 6x \div 3$

2 Simplify:

a $\frac{x}{2} + \frac{x}{3}$ **b** $\frac{2a}{5} - \frac{a}{10}$
c $\frac{3a}{2} \times \frac{5b}{6}$ **d** $\frac{10y}{3} \div 5y$
e $\frac{7x}{5} - \frac{x}{3}$ **f** $\frac{3m}{5} + \frac{m}{3} - \frac{m}{2}$
g $\frac{6n}{5} \times \frac{10}{7n} \div \frac{3}{2n}$ **h** $\frac{x+3}{2} + \frac{x+1}{3}$
i $\frac{2a-1}{5} + \frac{3a-2}{10}$ **j** $\frac{2n-1}{6} - \frac{2-n}{9}$

3 Factorise fully:

a $3a + 15$ **b** $6m + 9$
c $15 - 5y$ **d** $ax - 3x$
e $2x + 6xy$ **f** $4x^2 - 2x$
g $9ab - 6bc$ **h** $6x^2 - 9x + 3xy$

4 Expand and simplify:

a $x(x - 2)$
b $x - 2(x - 2)$
c $(x - 2)(x - 2)$
d $(x - 2)(x + 2)$
e $(x + 2)^2$
f $(2 - x)^2$

5 Expand and simplify where possible.

a $(x - 1)(x + 2)$
b $5x + 3(x - 1)$
c $2(x + 3) - 2x - 3$
d $(2x + 1)(x - 7)$
e $(x + 5)(x - 5)$
f $(3x + 2)^2$
g $x(x - 3) + 2(x + 1)$
h $(2 - x)(3 - x)$
i $(x + y)(y - x)$
j $(2x - y)^2$
k $5[x + 3(x + 1)]$
l $[3x - (x - 2)]^2$

If the width of one figure is x, what is the width of:
a 2 figures
b 4 figures
c half a figure?

Teacher's notes

Answers

ASSIGNMENT 3A

1 **a** $7a$ **b** $18x^2$
c $-4a$ **d** $2x^2$
e 6 **f** $\frac{3y}{2}$
g $2x + 3y$ **h** $6ab^2$
i $2ab$ **j** $12ab$
k $6a^2 - a$ **l** $-x - 3y$
m $19 + 5x$ **n** $12x$
o $4x^2 - x$ **p** $10x$

2 **a** $\frac{5x}{6}$ **b** $\frac{3a}{10}$
c $\frac{5ab}{4}$ **d** $\frac{2}{3}$
e $\frac{16x}{15}$ **f** $\frac{13m}{30}$
g $\frac{8n}{7}$ **h** $\frac{5x+11}{6}$
i $\frac{7a-4}{10}$ **j** $\frac{8n-7}{18}$

3 **a** $3(a + 5)$ **b** $3(2m + 3)$
c $5(3 - y)$ **d** $x(a - 3)$
e $2x(1 + 3y)$ **f** $2x(2x - 1)$
g $3b(3a - 2c)$ **h** $3x(2x - 3 + y)$

4 **a** $x^2 - 2x$ **b** $-x + 4$
c $x^2 - 4x + 4$ **d** $x^2 - 4$
e $x^2 + 4x + 4$ **f** $4 - 4x + x^2$

5 **a** $x^2 + x - 2$ **b** $8x - 3$
c 3 **d** $2x^2 - 13x - 7$
e $x^2 - 25$ **f** $9x^2 + 12x + 4$
g $x^2 - x + 2$ **h** $6 - 5x + x^2$
i $y^2 - x^2$ **j** $4x^2 - 4xy + y^2$
k $20x + 15$ **l** $4x^2 + 8x + 4$

Teaching strategies

Question 2

When students have completed Question 2 of Assignment 3A, ask them to explain the common errors that occur when working with algebraic fractions. This will enhance their understanding of the concepts.

Assessment

Assignment 3A

It is important for students to show their working. Award marks for working when it is applicable. Students could submit the assignment as an assessment task.

Teaching strategies

Assignment 3B

This assignment will help develop problem-solving skills and mathematical techniques. As a group, research other census data that has been graphed. Give students an opportunity to analyse graphs before completing the assignment.

Technology

Census data

Search the internet for 'graphed census data'. Create your own questions on the graphed census data. Present the questions to the class.

Worked solutions

ASSIGNMENT 3B

1 a square
b rectangle
c parallelogram
d rhombus
e trapezium
f pentagon
g hexagon
h octagon
i kite
j isosceles triangle

2 a An octagonal prism
b Sample answer: 200 mL

3 $2 + 3 \times 2$ (children and spouses) $+ 3 \times 3$ (grandchildren) $= 17$

4 a 4 (1 and 9, 2 and 8, 3 and 7, 4 and 6)
b 10 (make a list)

5 a 4 out of 10 = 40%
b 0·5 out of 10 = 5%

6 a Tasmania; 60% (the highest purple bar on the left)
b Victoria; over 90% (the highest purple bar on the right)
c Queensland, just under 3000 per 10 000
d About 50%; between 30% and 40%.

ASSIGNMENT 3B Working mathematically

1 Refer to **ID Card 4** on page xxi to identify the mathematical terms numbered:
a 1 b 2 c 3
d 4 e 5 f 6
g 7 h 8 i 9
j 11

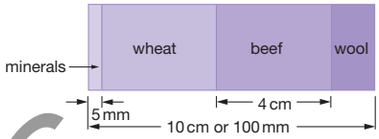
2 a What geometric shape has inspired the design of these coffee cups?

b What would you estimate the capacity of the cup to be?

3 Diane and Garry married and had three children. Each child married and had three children. Assuming that no one has died, how many people are now in this extended family altogether?

4 The numerals 1 to 10 are written on ten separate cards, one on each card.
a How many pairs of cards have a sum of 10?
b How many groups of three cards are there that have a sum of 18?

5 A particular country's exports are shown in the bar graph below (reduced in size). Find what percentage of the country's exports are taken up by:
a beef
b minerals.



6 Education of children, ages 5 to 14

Source: 1861 and 1921 Censuses

a In 1861, which state had the greatest number per 10 000 children who could read and write? What percentage was this?
b In 1921, which state had the greatest percentage of children who could read and write? What percentage was this?
c Which state had 4000 per 10 000 children who could read and write in 1861? About how many in that state could not read in 1861?
d Consider Western Australia in 1861. Approximately what percentage could read and write? Approximately what percentage could not read? (To determine this, measure the height of this column and measure this height on the scale.)

Teacher's notes

ASSIGNMENT 3C Cumulative revision

1 a Increase a wage of $900 by 6·5%. 1:01, 2:01
b In 1980 the population of a town was 42 000. By 2010 the population had decreased by 3·5%. What was the population of the town at that time?
c Which is larger?
i 15% or 20% ii $\frac{5}{12}$ or $\frac{2}{5}$ iii 0·65 or 0·0655

2 Change the following to decimals. 1:01
a $\frac{7}{8}$ b 6·8% c $\frac{5}{12}$

3 State the number of significant figures in the following measurements. 1:05
a 8·8 L b 123·45 m c 6 km d 4·0°C

4 In what range would each of the measurements in Question 3 lie? 1:10

5 Estimate the answers to the following calculations. 1:06, 1:07
a $\frac{9{\cdot}89 \times 3{\cdot}123}{5{\cdot}089}$ b $4{\cdot}95^2 + 2{\cdot}13 \times 5{\cdot}237$
c $\frac{212{\cdot}3}{\sqrt{104{\cdot}2}}$ d $\frac{48{\cdot}24 \times 0{\cdot}888}{(12{\cdot}5 - 7{\cdot}056)^2}$

6 Convert the following. 1:09
a 3·2 ML to L b 45 GW to W
c 3TB to MB d 15 MHz to kHz

7 Car A uses petrol at the rate of 4·8 L/100 km, whereas car B uses it at a rate of 10 L/100 km. What will the difference in petrol costs be in a year in which both cars travel 30 000 km, if the petrol costs on average $1.45/L? 1:04, 2:01

8 a A greengrocer buys 20 cases of oranges at a cost of $15 per case. Each case contains 10 kg of oranges. If he sells the oranges at $4/kg, how many kilograms must he sell before he makes a profit? If he sells all the oranges what will be his profit? 2:01
b Concrete is made by mixing volumes of cement, sand and gravel in the ratio 1 : 4 : 5. Jim calculates that he needs 2 m^3 of concrete to finish a job. How many cubic metres of sand and gravel does he need to order to make the concrete?

9 What is the last digit of the number 2^{2014}? 2:02

10 How many pairs of parallel edges are there in a rectangular prism? 2:02

Assessment

Assignment 3C

Use the cumulative revision as exam practice. Ensure that marks are awarded for working. Keep track of cumulative revision scores to check on improvement.

Answers

ASSIGNMENT 3C

1 a $958.50
b 40 530
c i 20%
ii $\frac{5}{12}$
iii 0·65

2 a 0·875
b 0·068
c $0.41\dot{6}$

3 a 2
b 5
c 1
d 2

4 a 8·75 m up to but not including 8·85
b 123·445 up to but not including 123·455
c 5·5 km up to but not including 6·5 km
d 3·95°C up to but not including 4·05°C

5 a 6
b 35
c 21
d 2

6 a 3 200 000 L
b 45 000 000 000 W
c 3 000 000 MB
d 15 000 kHz

7 $2262

8 a 75kg; $500
b Volume of sand = 0·8 m^3
Volume of gravel = 1 m^3

9 4

10 18

Teacher's notes