PEARSON

Mathematics

STUDENT BOOK | VICTORIA

10–10A

VICTORIAN CURRICULUM

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COVER **Shutterstock**: Andreichyn, Yurii, geometric pattern; Empics, graphs; Joingate, Fibonacci pattern; Kseniya, Abramova, horse.

TOPIC 4

Linear inequalities

Why learn this?

Understanding how linear inequalities and their associated regions in the plane are represented is fundamental for several reasons:

- It forms the basis for visualising and solving linear inequalities, which are common in algebra and essential for calculus.
- Graphing regions helps with understanding the relationship between variables and their constraints within a given system. This is important in fields such as economics, physics and engineering, where more than one solution may exist and it is necessary to identify the optimal one.

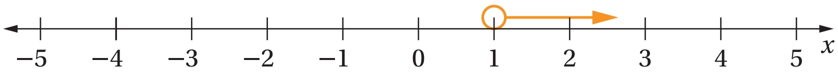
Graphing regions helps to visualise the possible and impossible solutions to mathematical problems. This supports analytical thinking and decision-making in academic and professional settings.

RECALL

I can interpret inequalities displayed on a number line

1 Write the inequality represented by each of the following.

(a)



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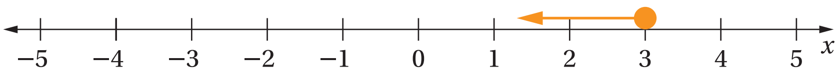

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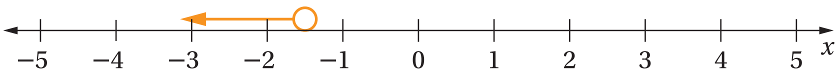

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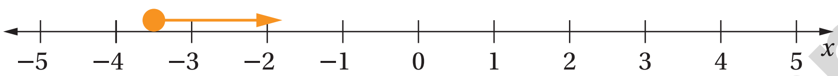

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I can graph linear equations

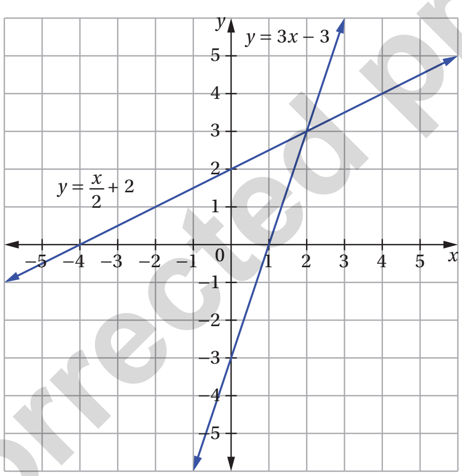
1 Graph each of the following linear equations on the same Cartesian number plane.

(a) $y = x - 4$ **(b)** $y = -2x + 1$ **(c)** $2y = x + 4$ **(d)** $3x - y = 3$

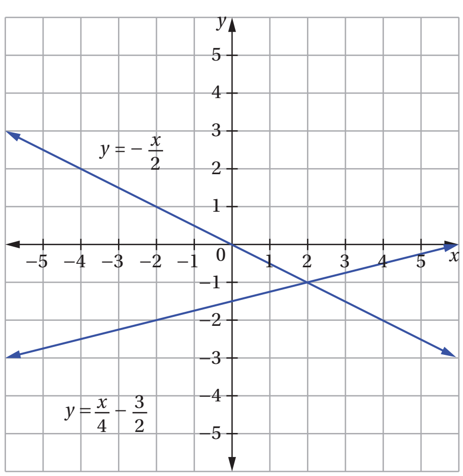
I can identify the mutual solution of two linear equations graphically

1 Identify the coordinates of the mutual solution for each pair of graphed linear equations.

(a)



(b)



I can solve simultaneous linear equations algebraically

1 Solve the following pairs of simultaneous linear equations.

(a) $x + y = 4$ and $2x - y = 8$

(b) $3x - 2y = 5$ and $x - 2y = 3$

(c) $2x - y = 10$ and $2x + 3y = 6$

(d) $4x - 3y = 2$ and $4y - 4x = 3$

Graph vertical and horizontal regions in the plane

4.1

Learning intention: To be able to graph vertical and horizontal regions in the plane.

Success criteria:

SC 1 I can graph horizontal and vertical regions with single boundaries.

SC 2 I can graph regions based on a horizontal or vertical interval.

SC 3 I can graph regions with horizontal and vertical boundaries.

Lesson warm-up

The divider

Practice using the language of less than, equal to and greater than to describe the relationship between everyday classroom objects.

Compare the width, length or weight of two items using statements such as 'is less than', 'is equal to' or 'is greater than.'

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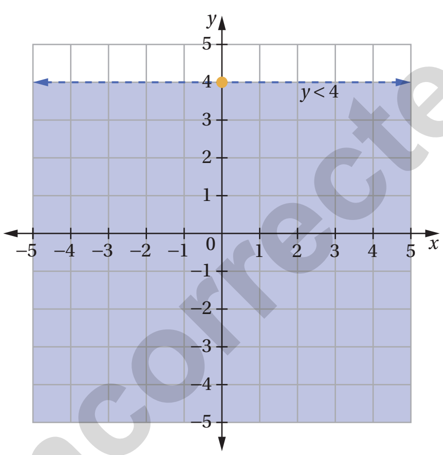
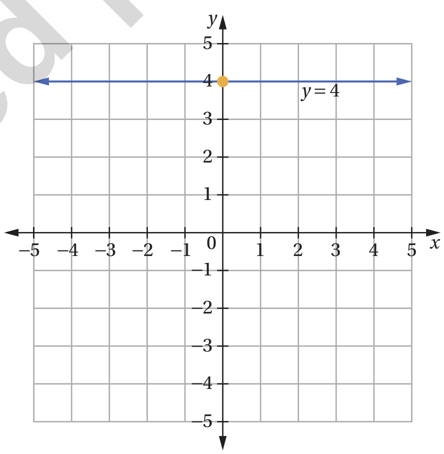
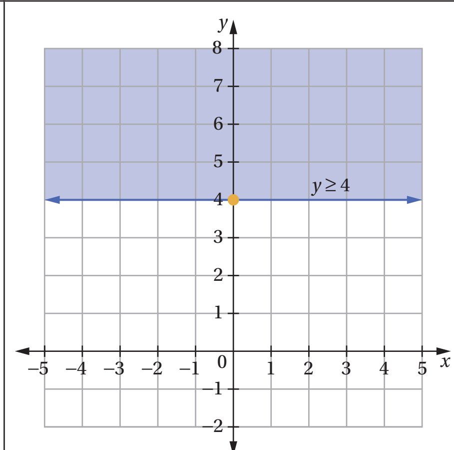
SC 1 I can graph horizontal and vertical regions with single boundaries

$y < 4$	$y = 4$	$y \geq 4$
		
A dashed line has been used as the boundary, but it is not included in the region. The region containing points with y-values of less than 4 has been shaded.	A solid line has been used for the graph. No region is shaded, as the points with a y-value of 4 all lie on the line.	A solid line has been used as the boundary and is included in the region. The region containing points with y-values greater than or equal to 4 has been shaded.

When there is an inequality sign, the graph of the inequality is a shaded region.

A region is shaded when it contains the set of points that satisfies the inequality. Often it is easier to test a point, for example, (0, 0), to determine which side of the boundary to shade. A solid boundary line indicates a line that satisfies the inequality.

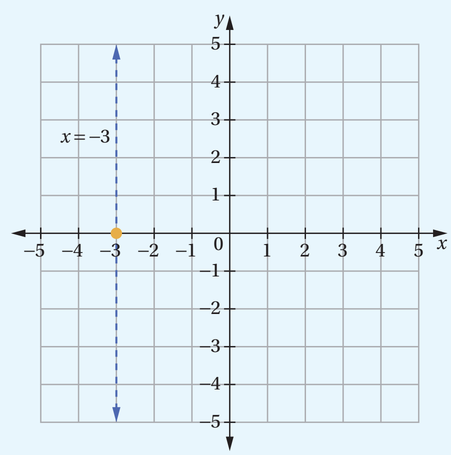
Symbol	Meaning	Type of boundary line
$<$ or $>$	less than or greater than	Dashed line
$\leq$ or $\geq$	less than or equal to or greater than or equal to	Solid line

Worked example

Graphing horizontal and vertical regions using the less than and greater than symbols

Sketch a graph of the following regions.

(a) $x > -3$

THINKING	WORKING
Determine whether the boundary line will be solid or dashed.	The inequality sign is greater than (>), so the boundary line is dashed, meaning it is not included in the region.
Sketch the boundary line.	The boundary line is $x = -3$. 
Recall the meaning of the inequality symbol.	$x > -3$ means that the shaded region should cover the x-values that are greater than -3.

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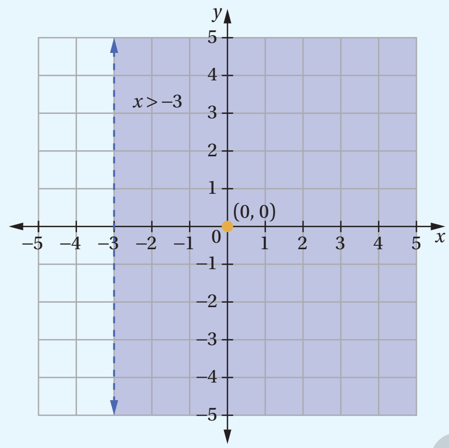
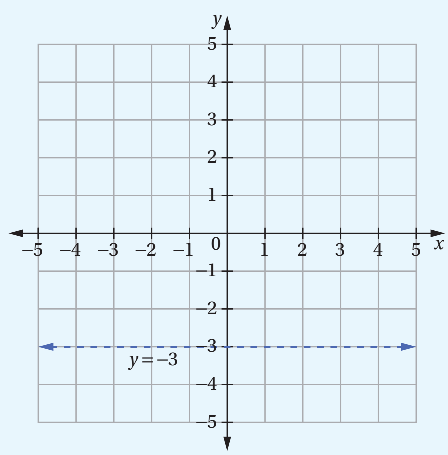
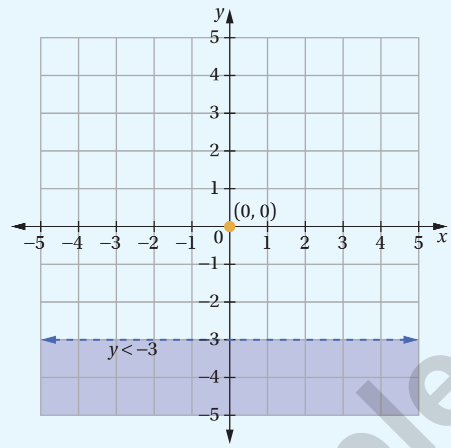
Shade and label the region.	
Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	At the origin (0, 0), $x = 0$: $0 > -3$. This is true; therefore, the point satisfies the inequality and lies in the shaded region.

(b) $y < -3$

THINKING	WORKING
Determine whether the boundary line will be solid or dashed.	The inequality sign is less than (<), so the boundary line is dashed, meaning it is not included in the region.
Sketch the boundary line.	The boundary line is $y = -3$. 
Recall the meaning of the inequality symbol.	$y < -3$ means that the shaded region should cover the y-values that are less than -3.

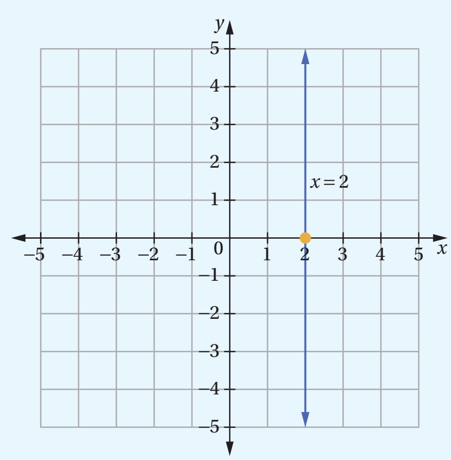
Shade and label the region.	
Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	For the test point (1, −4), $y = -4$ and $-4 < -3$. This is true; therefore, the point satisfies the inequality and lies in the shaded region.

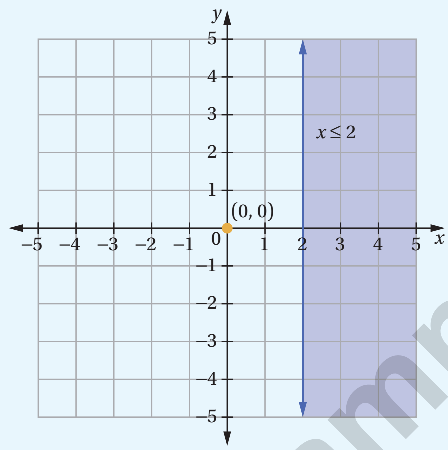
Worked example

Graphing horizontal and vertical regions using the less than or equal to and greater than or equal to symbols

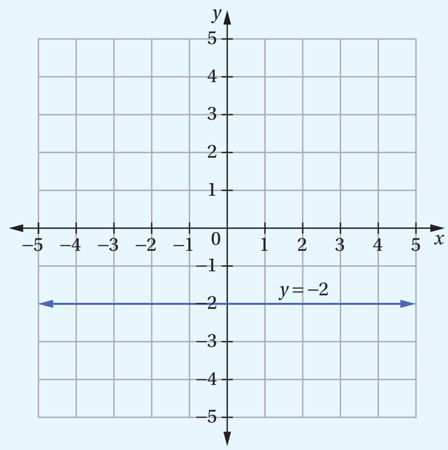
Sketch a graph of each of the following regions.

(a) $x \geq 2$

THINKING	WORKING
Determine whether the boundary line will be solid or dashed.	The inequality sign is greater than or equal to (≥), so the boundary line is solid, meaning it is included in the region.
Sketch the boundary line.	The boundary line is $x = 2$. 

Recall the meaning of the inequality used.	$x \geq 2$ means that the shaded region should cover the x-values that are greater than or equal to 2.
Shade and label the region.	
Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	For the test point (3, 1), $x = 3$ and $3 \geq 2$. This is true; therefore, the point satisfies the inequality and lies in the shaded region.

(b) $y \leq -2$

THINKING	WORKING
Determine whether the boundary line will be solid or dashed.	The inequality sign is greater than or equal to ($\geq$), so the boundary line is solid, meaning it is included in the region.
Sketch the boundary line on a graph. Check to see whether the boundary line is included in the region. Does the inequality sign also include 'equal to'?	The boundary line is $y = -2$. The inequality sign is less than or equal to ($\leq$), so the boundary line is solid, meaning it is included in the region. 

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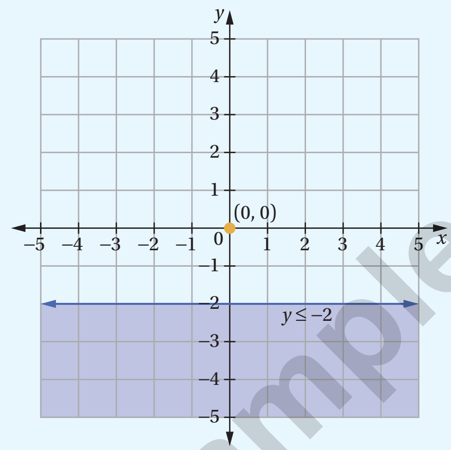
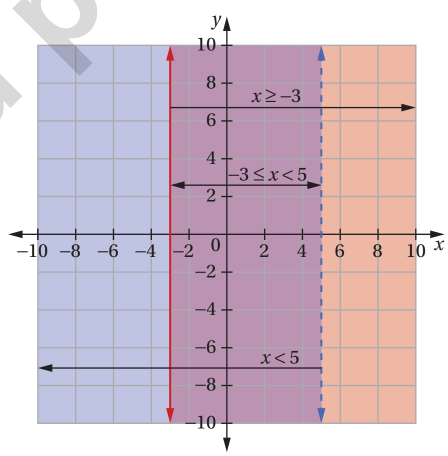
<table>
<tr><td>Recall the meaning of the inequality used.</td><td>$y \le -2$ means that the shaded region should cover the y-values that are greater than or equal to -2.</td></tr>
<tr><td>Shade and label the region.</td><td>
</td></tr>
<tr><td>Substitute the x- and y-values of a test point to verify that the shaded region is accurate.</td><td>For the test point $(2, -3)$, $y = -3$ and
$-3 \le -2$.
This is true; therefore, the point satisfies the inequality and lies in the shaded region.</td></tr>
</table>

SC 2 I can graph regions based on a horizontal or vertical interval

When an interval of values is sketched on the same set of axes, an intersecting region is formed. The set of points inside the intersecting region will be the solution to the inequalities.



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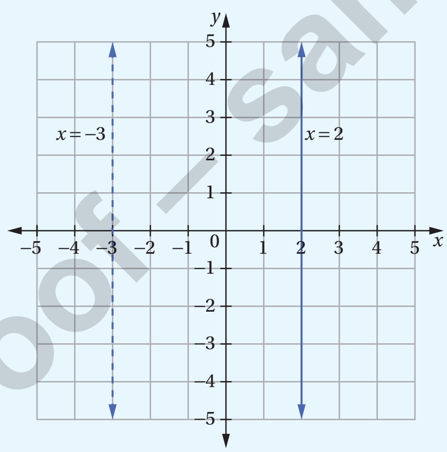
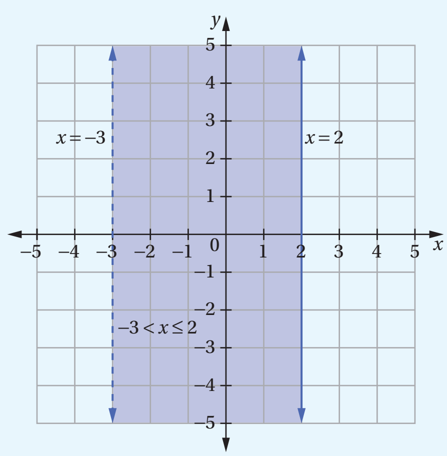
Worked example

Graphing horizontal and vertical intervals

Sketch the following regions on the Cartesian plane.

(a) $-3 < x \leq 2$

THINKING	WORKING
Identify and sketch the boundary lines.	Boundary line 1: $-3 < x$ is the same as $x > -3$. The boundary at $x = -3$ is dashed. Boundary line 2: For $x \leq 2$, the boundary at $x = -2$ is solid. 
Shade the region.	
Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	Substituting $x = 0$ results in $-3 < 0 \leq 2$. This creates a true interval; therefore, the point lies within the interval.

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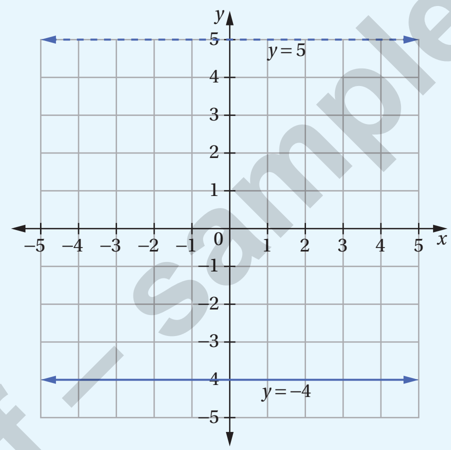
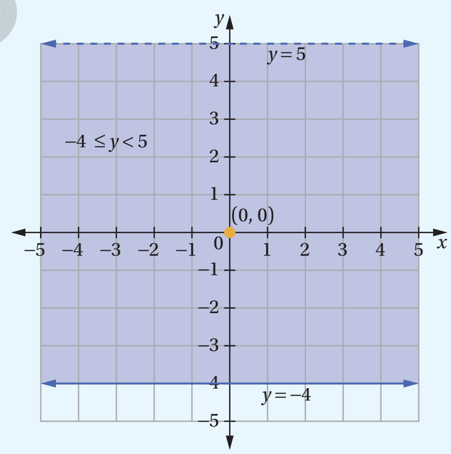
(b) $-4 \leq y < 5$

THINKING	WORKING
Identify and sketch the boundary lines.	Boundary line 1: $-4 \leq y$ is the same as $y \geq -4$. The boundary at $x = -4$ is solid. Boundary line 2: For $y < 5$, the boundary at $y = 5$ is solid. 
Shade the region.	
Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	Substituting $y = 0$ results in $-4 \leq 0 < 5$. This creates a true interval; therefore, the point lies within the interval.

4

SC 3 I can graph regions with horizontal and vertical boundaries

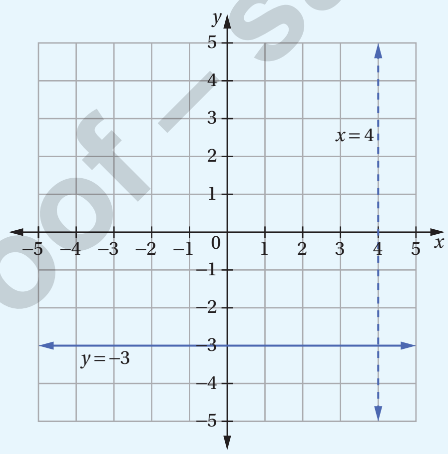
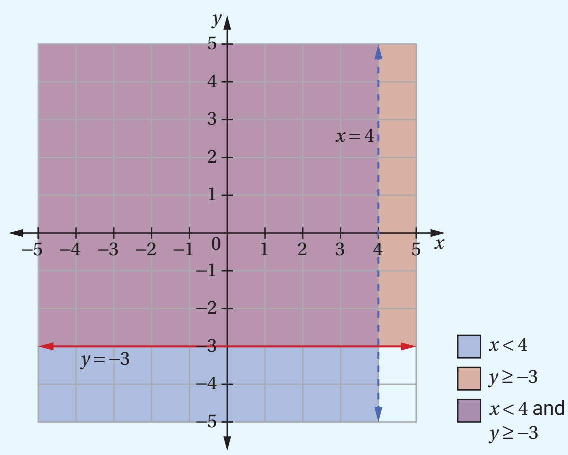
As the number of inequalities increases, the region that satisfies all of them becomes more restricted.

Worked example

Graphing regions with both horizontal and vertical boundaries

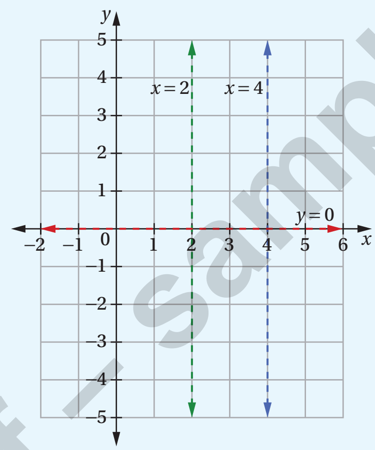
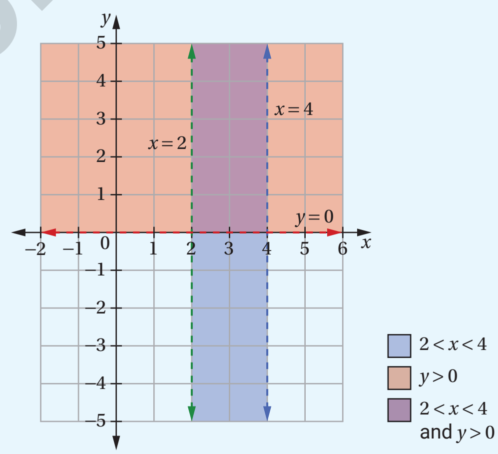
Sketch each of the following on separate sets of Cartesian axes.

(a) $x < 4$ and $y \geq -3$

THINKING	WORKING
Identify and sketch the boundary lines.	Region 1: $x < 4$ The boundary line at $x = 4$ is dashed. Region 2: $y \geq -3$ The boundary line at $y = -3$ is solid. 
Shade and label each of the regions.	
Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	For the origin (0, 0), $x = 0$ and $y = 0$ satisfy each inequality. $0 < 4$ and $0 \geq -3$; therefore, the origin is within the shaded region.

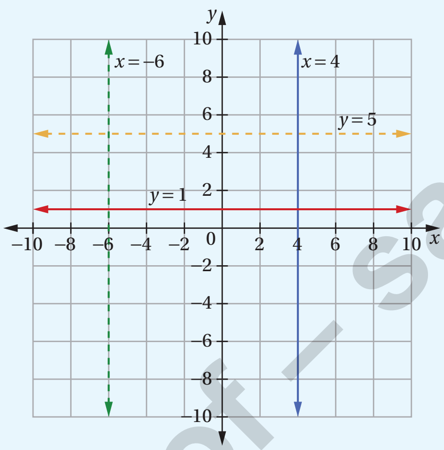
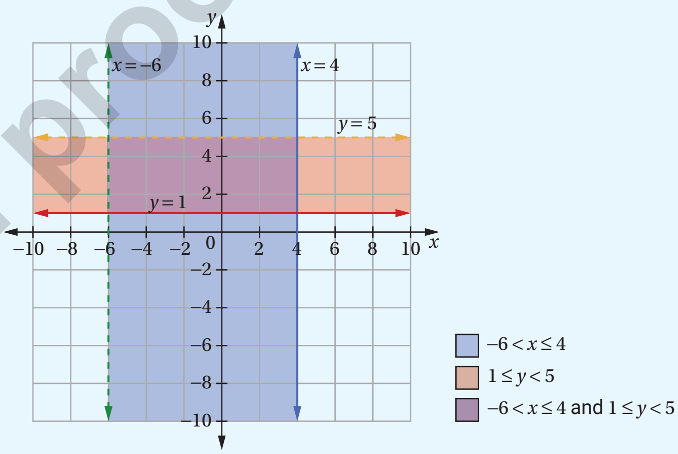
4

(b) $2 < x < 4$ and $y > 0$

THINKING	WORKING
Identify and sketch the boundary lines.	Boundary line 1: $2 < x < 4$ The boundary lines at $x = 2$ and $x = 4$ are dashed. Region 2: $y > 0$ The boundary line at $y = 0$ is dashed. 
Shade and label each of the regions.	
Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	Regarding the point (3, 1), x: $2 < 3 < 4$ y: $1 > 0$ The point (3, 1) is within the shaded region.

4

(c) $-6 < x \leq 4$ and $1 \leq y < 5$

THINKING	WORKING
Identify and sketch the boundary lines.	Boundary line 1: $-6 < x \leq 4$ The boundary line at $x = -6$ is dashed, and at $x = 4$ it is solid. Region 2: $1 \leq y < 5$ The boundary line at $y = 1$ is solid, and the line at $y = 5$ is dashed. 
Shade and label each region.	
Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	For the point (0, 3), x: $-6 < 0 \leq 4$ y: $1 \leq 3 < 5$ Both statements are true; therefore, the point satisfies the region bounded by the inequality.

Practice

ANSWERS Page XXX

SC 1 I can graph horizontal and vertical regions with single boundaries

1 Sketch each of the following inequalities on a separate graph.

(a) $y > 3$ **(b)** $y < 3$ **(c)** $x < 3$ **(d)** $x > 3$

2 Sketch each of the following inequalities on a separate graph.

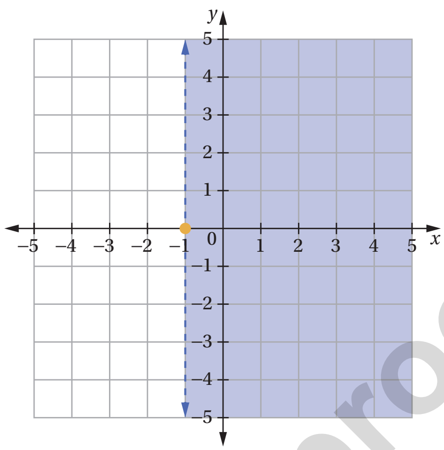
(a) $y \geq -1$ **(b)** $y \leq -1$ **(c)** $x \geq -1$ **(d)** $x \leq -1$

3 Sketch each of the following inequalities on a separate graph.

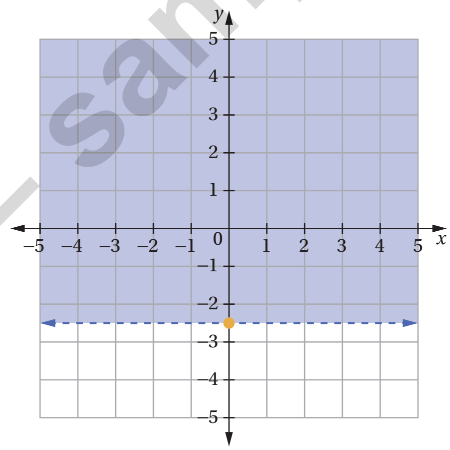
(a) $x \leq \frac{3}{2}$ **(b)** $y > -3.2$

4 Consider the following graphs of inequalities and determine the rule that best represents the inequality shown.

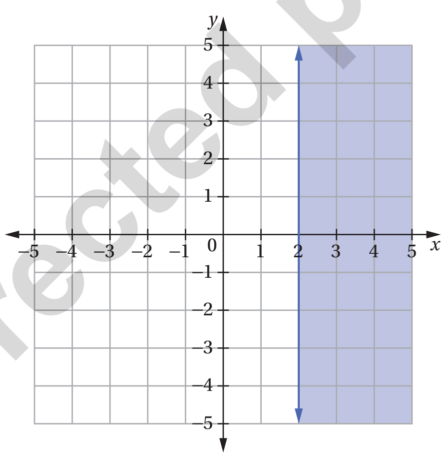
(a)



(b)



(c)



5 List any three pairs of coordinates that satisfy each of the following inequalities.

(a) $-3 \leq x$ **(b)** $y > \frac{5}{3}$ **(c)** $y \geq 4$

SC 2 I can graph regions based on a horizontal or vertical interval

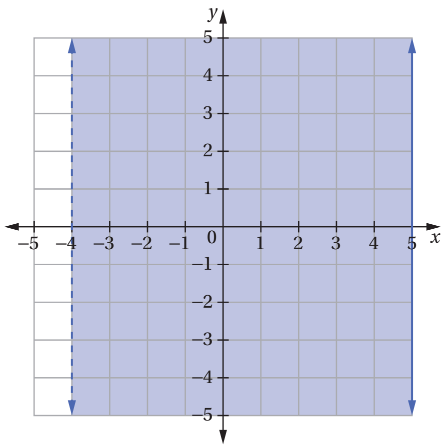
1 Sketch the following inequalities.

(a) $-3 \leq x < 5$ **(b)** $-3 < x \leq 5$ **(c)** $-3 \leq x \leq 5$ **(d)** $-3 < x < 5$

2 Sketch the following inequalities.

(a) $-2 < y < 3$ **(b)** $-2 < y \leq 3$ **(c)** $-2 \leq y < 3$ **(d)** $-2 \leq y \leq 3$

3 Consider the following graph.



(a) Write the interval shown.

(b) Select the points that belong to the interval from the list: (–4, 0), (2, 10), (–5, 4), (6, 2), (5, –2), (2, –4)

SC 3 I can graph regions with horizontal and vertical boundaries

1 For each of the following, state whether the given point lies in the region specified by the inequalities.

(a) $4 \leq x$ and $y < 2$; (5, 0)

(b) $x < 3$ and $0 \leq y \leq 8$; (3, 0)

(c) $-4 < x \leq 6$ and $-1 \leq y < 2$; (2, 2)

2 Sketch each of the following on separate sets of Cartesian axes.

(a) $x \leq 5$ and $y < 3$

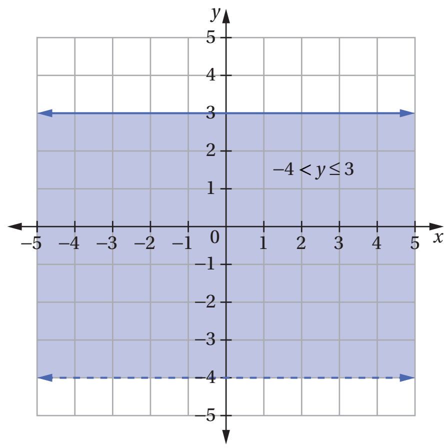
(b) $x \leq 3$ and$-3 < y \leq 4$

(c) $0 < x \leq 5$ and $-5 \leq y < 1$

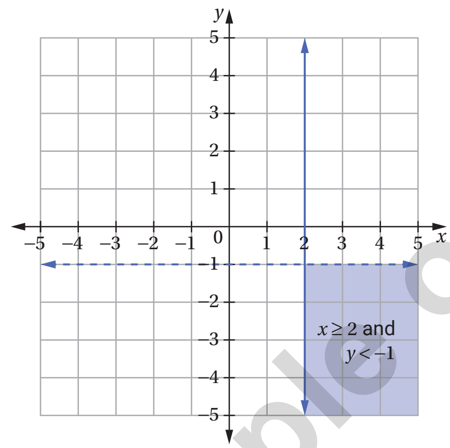
4

3 Match the graph of each of the following regions with the inequalities that define it.

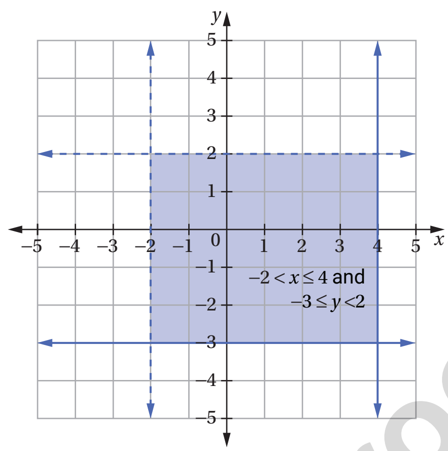
(a)



(b)



(c)



4 Calculate the area of the interior rectangle bounded by the inequalities $-3 \le x \le 2$ and $-3 \le y \le 2$.

4

Graph and describe linear inequalities with two variables in the plane

4.2

Learning intention: To be able to graph and describe linear inequalities with two variables in the plane.

Success criteria:

SC 1 I can graph linear inequalities with variables on different sides of the inequality symbol.

SC 2 I can graph linear inequalities with variables on the same side of the inequality symbol.

4

Lesson warm-up

Sometimes true, always true or never true?

Decide whether the following statements are sometimes true, always true or never true.

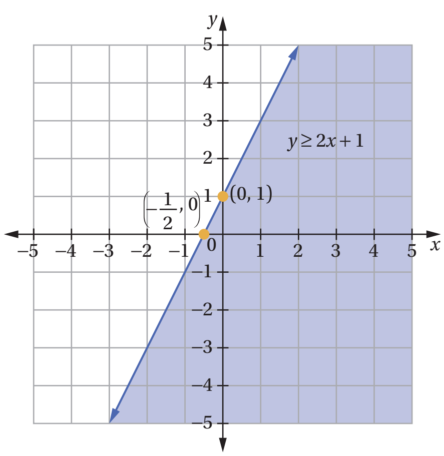
Explain your reasoning.

$-x$ is less than x

x^2 is greater than x

SC 1 I can graph linear inequalities with variables on different sides of the inequality symbol

The inequality $y \geq 2x + 1$ is an example of a linear inequality with variables on either side of the inequality symbol.

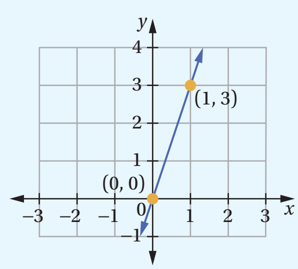
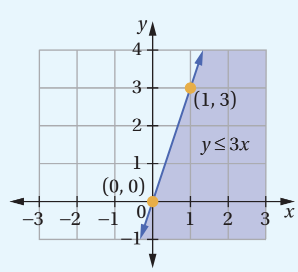


Worked example

Graphing linear inequalities with variables on each side of the inequality symbol

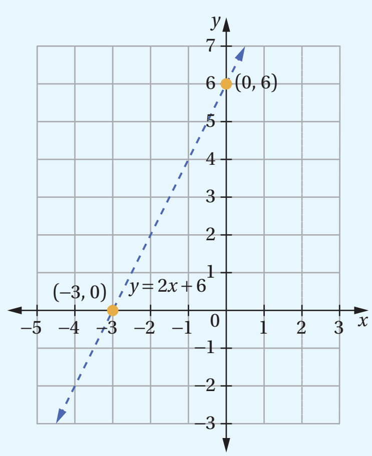
Sketch the graph of each of the following inequalities.

(a) $y \leq 3x$

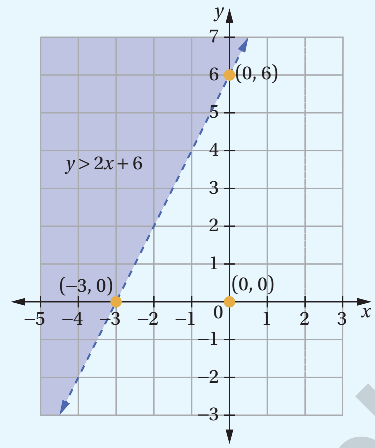
THINKING	WORKING
Determine the coordinates of the intercepts (or two points) of the straight line boundary.	Boundary line: $y = 3x$ x-intercept, let $y = 0$ $0 = 3x$ $0 = x$ The graph passes through the origin (0, 0). Additional point: Let $x = 1$ $y = 3 \times 1$ $= 3$ The coordinates of an additional point are (1, 3).
Determine whether the boundary line is dashed or solid.	$y \leq 3x$ The boundary line is included because the symbol is 'less than or equal to'; therefore, it is solid.
Plot the points on the Cartesian plane and join with the boundary line.	
Shade and label the region. A region that is 'less than' (<) or 'less than or equal to' (≤) will be shaded below the boundary line. A region that is 'greater than' (>) or 'greater than or equal to' (≥) will be shaded above the boundary line.	

Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	Check (1, 2) at $x = 1$ and $y = 2$. $2 \leq 3 \times x$ $2 \leq 3 \times 1$ $2 \leq 3$ The inequality is true; therefore, the point is in the shaded region.

(b) $y > 2x + 6$

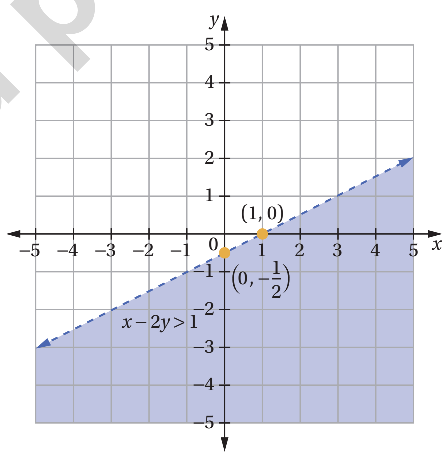
THINKING	WORKING
Determine the coordinates of the intercepts (or two points) of the straight line boundary. Determine the intercepts for the boundary line. Check to see whether the boundary line is included in the region.	Boundary line: $y = 2x + 6$ x-intercept, let $y = 0$ $0 = 2x + 6$ $-6 = 2x$ $x = -3$ The coordinates of the x-intercept are $(-3, 0)$. y-intercept, let $x = 0$ $y = 2 \times 0 + 6$ $= 6$ The coordinates of the y-intercept are (0, 6). The line will be dotted as the inequality is >; therefore, the boundary line is not included in the region.
Determine whether the boundary line is dashed or solid.	$y > 2x + 6$ The boundary line is not included in the region because the symbol is 'greater than'; therefore, it is dashed.
Plot the points on the Cartesian plane and join with the boundary line.	

4

Shade and label the region. A region that is 'less than' < or 'less than or equal to' ≤ will be shaded below the boundary line. A region that is 'greater than' > or 'greater than or equal to' ≥ will be shaded above the boundary line.	
Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	Check (−3, 4) at $x = -3$ and $y = 4$. $y > 2x + 6$ $4 > 2 \times (-3) + 6$ $4 > -6 + 6$ $4 > 0$ The inequality is true; therefore, the point is in the shaded region.

SC 2 I can graph linear inequalities with variables on the same side of the inequality symbol

The inequality $x - 2y > 1$ is an example of a linear inequality that has both variables on the same side of the inequality symbol.

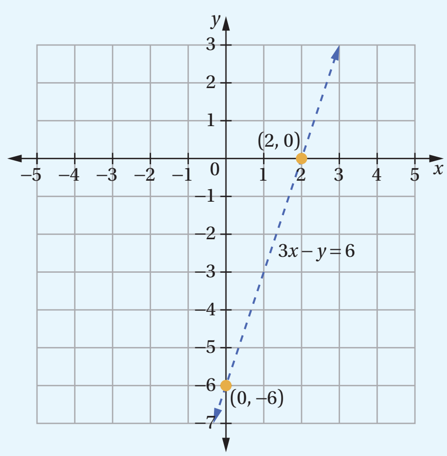


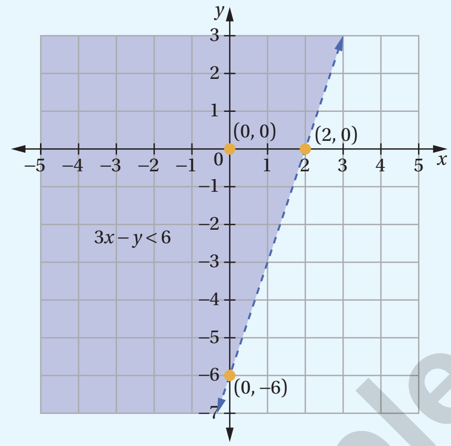
4

Worked example

Graphing linear inequalities with variables on the same side of the inequality symbol

Sketch the graph of $3x - y < 6$.

THINKING	WORKING
Determine the coordinates of the intercepts (or two points) of the straight line boundary.	Boundary line: $3x - y = 6$ y-intercept, let $x = 0$ $3 \times 0 - y = 6$ $0 - y = 6$ $y = -6$ The coordinates of the y-intercept are $(0, -6)$. x-intercept, let $y = 0$ $3x - 0 = 6$ $3x = 6$ $x = 2$ The coordinates of the x-intercept are $(2, 0)$.
Determine whether the boundary line is dashed or solid.	The boundary line will be dashed, and it is not included in the region because the inequality sign is $<$.
Plot the points on the Cartesian plane and join with the boundary line.	

Shade and label the region. A region that is 'less than' (<) or 'less than or equal to' (≤) will be shaded below the boundary line. A region that is 'greater than' (>) or 'greater than or equal to' (≥) will be shaded above the boundary line.	
Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	For (0, 0), $x = 0$ and $y = 0$ $3x - y < 6$ $3(0) - 0 < 6$ $0 < 6$ This is true; therefore, the point satisfies the inequality. The required region is on this side of the boundary line.

Practice

ANSWERS Page XXX

SC 1 I can graph linear inequalities with variables on different sides of the inequality symbol

1 Sketch each of the following inequalities on a separate graph.

(a) $y < x$ **(b)** $y > -x$ **(c)** $y \leq -2x$ **(d)** $y \geq 2x$

2 Sketch each of the following inequalities on a separate graph.

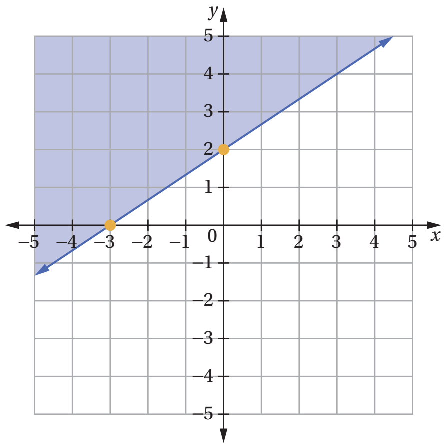
(a) $y > 2x + 1$ **(b)** $y \geq 2x + 1$ **(c)** $y < 2x + 1$ **(d)** $y \leq 2x + 1$

3 Sketch each of the following inequalities on a separate graph.

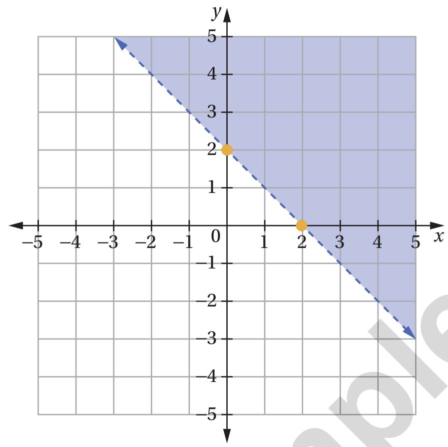
(a) $y > 4x$ **(b)** $4x \geq y$ **(c)** $y \leq \frac{1}{2}x + 2$ **(d)** $y > 6x + 3$

4 Consider the following graphs of inequalities and determine the rule that best represents the inequality shown.

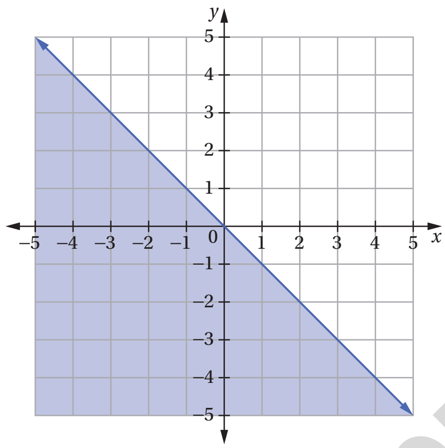
(a)



(b)



(c)



5 For each of the following, state whether the coordinate pair belongs in the area required for the given inequality.

(a) $y \leq 4 - x$; (1, –1)

(b) $y > \dfrac{3(x-2)}{2}$; (–2, –8)

(c) $y \geq \dfrac{5(-3x+2)}{3}$; (1, –3)

(d) $y < -3\left(\dfrac{6-2x}{4}\right)$; (3, –2)

SC 2 I can graph linear inequalities with variables on the same side of the inequality symbol

1 Consider the inequality $12x + 2y < 4$.

(a) Determine the coordinates of the y-intercept.

(b) Determine the coordinates of the x-intercept.

(c) Determine whether the boundary line is dashed or solid.

(d) Verify whether (0, 0) is in the shaded region or not.

(e) Graph the inequality.

2 For each of the following inequalities in the form $ax + by = c$, write in gradient-intercept form and then graph the inequality.

(a) $y + 3x < 8$ (b) $y + 3x \geq 8$ (c) $y - 3x \leq 8$ (d) $y - 3x > 8$

3 For each of the following inequalities in the form $ax + by = c$, write in gradient-intercept form and then graph the inequality.

(a) $2y + 6x < 9$ (b) $2y + 6x \geq 9$ (c) $2y - 6x > 9$ (d) $2y - 6x \leq 9$

4 Graph the regions $y > -x$ and $-y > x$. Use your answer to explain how a region specified with a greater than symbol can include shading under the straight line.

5 For each of the following inequalities in the form $ax + by = c$, write in gradient-intercept form and then graph the inequality.

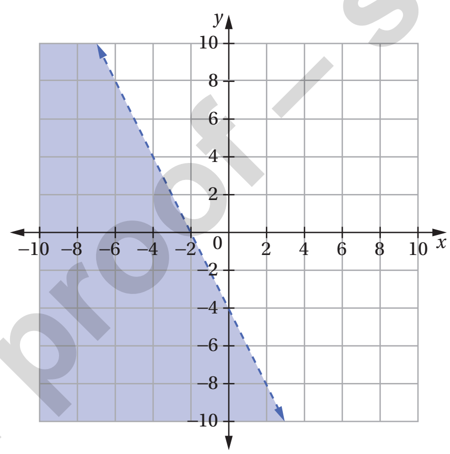
(a) $y - 2x > 1$ (b) $2x - y \geq 1$ (c) $y - 2x \leq -1$ (d) $2x - y < -1$

6 Sketch each of the following regions.

(a) $2y - 3x \geq -16$ (b) $y + 2x < -6$

4

7 Consider the following graph.



(a) Write the linear inequality that best describes the above graph.

(b) Using the graph, verify whether (–4, 2) is a solution to the inequality.

(c) Determine the points of two solutions to the inequality.

Graph simultaneous inequalities

Learning intention: To be able to graph simultaneous inequalities.

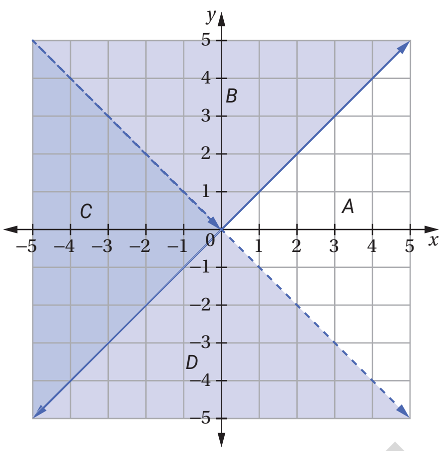
Success criteria:

SC 1 I can graph regions in the Cartesian plane using simultaneous inequalities.

Lesson warm-up

Intersecting regions

Use graphing software to sketch $y \geq x$ and $y < -x$ on the same graph.



Region A is unshaded, which indicates no solution lies within this region.

Region B is shaded once, which indicates this region satisfies one of the inequalities.

Region C is shaded once, which indicates this region satisfies one of the inequalities.

Region D is the overlapping region, which indicates the solution set to both inequalities.

Now, plot each of the following points on the same graph: (0, 1), (1, 0), (0, –1), (–1, 0) and (–3, 1).

What does the overlapping shaded region represent? Explain your reasoning.

SC 1 I can graph regions in the Cartesian plane using simultaneous inequalities

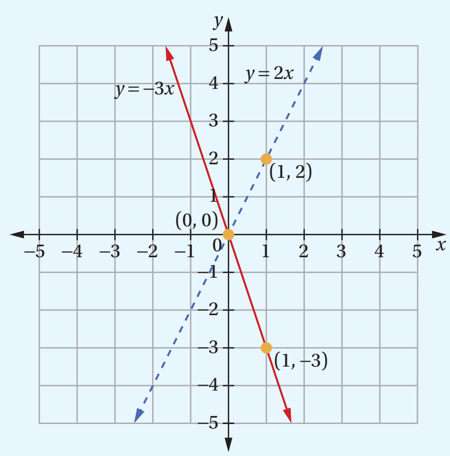
When two linear inequalities are sketched on the same set of axes, and the boundary lines intersect, an intersecting region is formed. The point of intersection of the boundary lines is the solution to the corresponding simultaneous equations. Points within the intersecting region are solutions to the system of linear inequalities. A test point such as (0, 0) can be used to identify or verify the intersection region so long as it is not on one of the boundary lines.

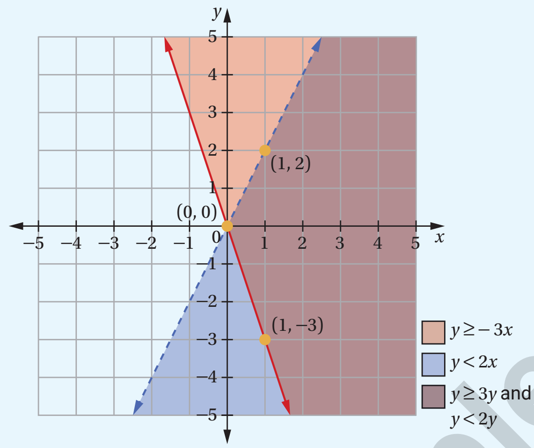
Worked example

Graphing regions using simultaneous inequalities

Shade the intersecting region for each of the following pairs of inequalities.

(a) $y \geq -3x$ and $y < 2x$

THINKING	WORKING
Determine the coordinates of the intercepts (or two points) of each of the straight-line boundaries.	Graphs in the form $y = ax$ pass through the origin (0, 0). Boundary line 1: $y = -3x$ At $x = 1$: $y = -3 \times (1)$ $= -3$ The coordinates of another point are (1, –3). Boundary line 2: $y = 2x$ At $x = 1$: $y = 2 \times (1)$ $= 2$ The coordinates of another point are (1, 2).
Determine whether the boundary lines are dashed or solid.	The line $y = -3x$ will be solid because the boundary line is included in the region ($\geq$). The line $y = 2x$ will be dashed since the boundary line is not included in the region ($<$).
Plot the points on the Cartesian plane and join with the boundary line.	

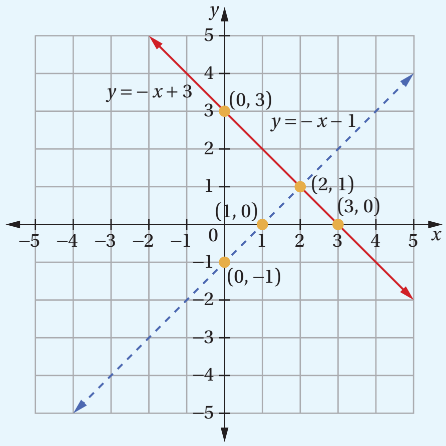
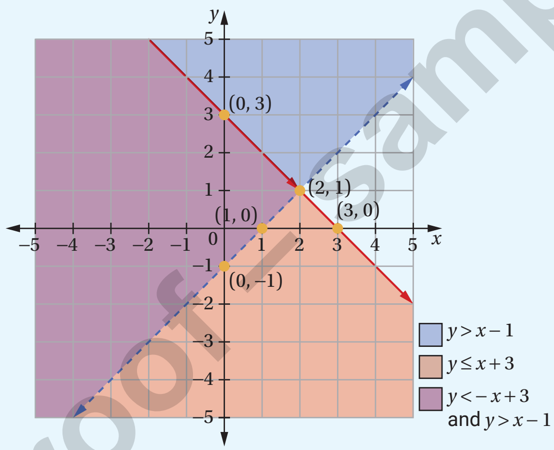
Shade and label the regions. A region that is 'less than' (<) or 'less than or equal to' (≤) will be shaded below the boundary line. A region that is 'greater than' (>) or 'greater than or equal to' (≥) will be shaded above the boundary line.	
Substitute the x- and y-values of a test point to verify the shaded region is accurate.	The test point is (2, 0). Substitute the test point into each of the inequalities. $y \geq -3x$ $0 \geq -3 \times 2$ $0 \geq -6$ $y < 2x$ $0 < 2 \times 2$ $0 < 4$ The inequalities are both true; therefore, the point is in the required region.

4

(b) $y \le -x + 3$ and $y > x - 1$

THINKING	WORKING
Determine the coordinates of the intercepts (or two points) of each of the straight-line boundaries.	Boundary line 1: $y = -x + 3$ y-intercept, let $x = 0$: $y = -0 + 3$ $= 0 + 3$ $= 3$ The coordinates of the y-intercept are (0, 3). x-intercept, let $y = 0$: $0 = -x + 3$ $x = 3$ The coordinates of the x-intercept are (3, 0). Boundary line 2: $y = x - 1$ y-intercept, let $x = 0$: $y = 0 - 1$ $= -1$ The coordinates of the y-intercept are (0, –1). x-intercept, let $y = 0$: $0 = x - 1$ $1 = x$ The coordinates of the x-intercept are (1, 0).
Determine whether the boundary lines are dashed or solid.	The boundary line $y = -x + 3$ will be solid because the boundary line is included in the region (≤). The line $y > x - 1$ will be dashed because the boundary line is not included in the region (>).
Determine the coordinates of the point of intersection.	$-x + 3 = x - 1$ $3 = 2x - 1$ $4 = 2x$ $2 = x$ Substitute the value of x into one of the equations of the boundary lines: $y = x - 1$ $= 2 - 1$ $= 1$ The coordinates of the point of intersection are (2, 1).

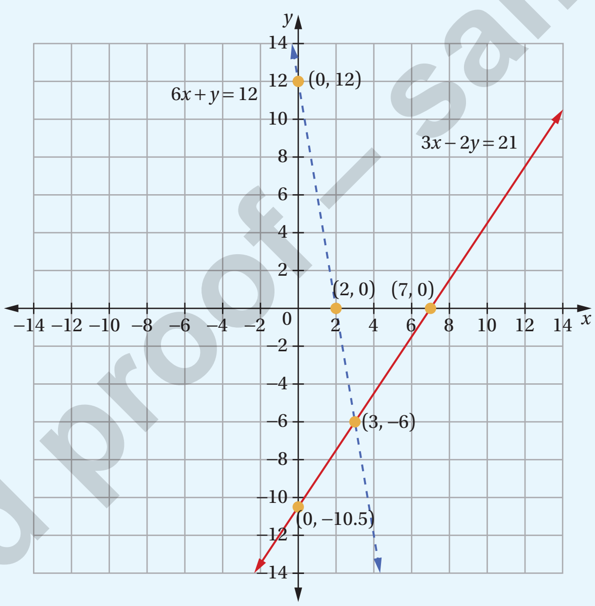
4

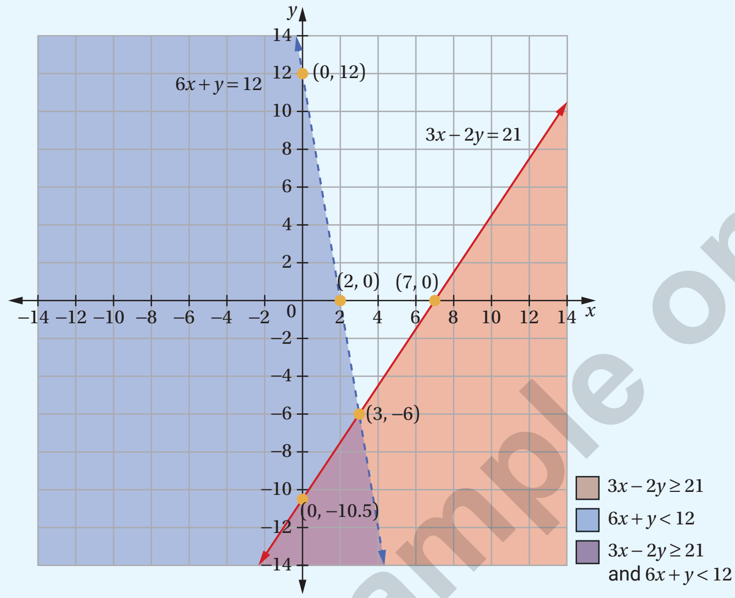
Plot the points on the Cartesian plane and join with the boundary line.	
Shade and label the regions. A region that is 'less than' (<) or 'less than or equal to' (≤) will be shaded below the boundary line. A region that is 'greater than' (>) or 'greater than or equal to' (≥) will be shaded above the boundary line.	
Substitute the x- and y-values of a test point to verify that the shaded region is accurate.	The test point is (0, 0). Substitute the test point into each inequality. $y \le -x + 3$ $0 \le -0 + 3$ $0 \le 3$ $y > x - 1$ $0 > 0 - 1$ $0 > -1$ The inequalities are both true; therefore, the point is in the required region.

4

(c) $6x + y < 12$ and $3x - 2y \geq 21$

THINKING	WORKING
Determine the coordinates of the intercepts (or two points) of each of the straight-line boundaries.	Boundary line 1: $6x + y = 12$ y-intercept, let $x = 0$:\\ $6 \times 0 + y = 12$ $0 + y = 12$ $y = 12$ The coordinates of the y-intercept are (0, 12). x-intercept, let $y = 0$: $6x + 0 = 12$ $6x = 12$ $x = 2$ The coordinates of the x-intercept are (2, 0). Boundary line 2: $3x - 2y = 21$ y-intercept, let $x = 0$: $3 \times 0 - 2y = 21$ $0 - 2y = 21$ $-2y = 21$ $y = -\frac{21}{2}$ or -10.5 The coordinates of the y-intercept are (0, −10.5). x-intercept, let $y = 0$: $3x - 2 \times 0 = 21$ $3x - 0 = 21$ $3x = 21$ $x = 7$ The coordinates of the x-intercept are (7, 0).
Determine whether the boundary lines are to be dashed or solid.	The boundary line $3x - 2y \geq 21$ will be solid because the boundary line is included in the region ($\geq$). The line $6x + y < 12$ will be dashed because the boundary line is not included in the region ($<$).

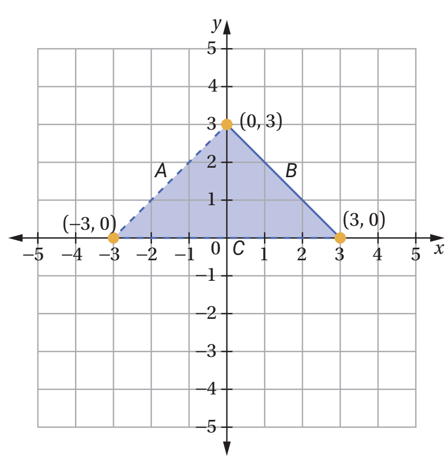
Determine the coordinates of the point of intersection.	Solve the simultaneous equations using the method of elimination. $6x+y=12 \quad [1]$ $3x-2y=21 \quad [2]$ $[1]\times 2 \Rightarrow \underline{12x+2y}=\underline{24} \quad [3]$ $[2]+[3] \Rightarrow \quad 5x=45$ $x=3$ Substitute $x = 3$ into $6x + y = 12$. $6 \times 3 + y = 12$ $18 + y = 12$ $y = -6$ The coordinates of the point of intersection are (3, −6).
Plot the points on the Cartesian plane and join with the boundary line.	
Write the inequalities in the form $y = mx + c$. Written in this form, the region that is ‘less than’ (<) or ‘less than or equal to’ (≤) will be shaded below the boundary line. A region that is ‘greater than’ (>) or ‘greater than or equal to’ (≥) will be shaded above the boundary line.	$6x + y < 12$ $y < 12 - 6x$ and $3x-2y \geq 21$ $3x \geq 21+2y$ $3x-21 \geq 2y$ $2y \leq 3x-21$ $y \leq \frac{3}{2}x-10.5$

Shade and label the regions.	
Substitute the x- and y-values of a test point to verify the shaded region is accurate.	The test point is (2, −14). Substitute the test point into the inequality. $6 \times 2 + (-14) < 12$ $12 - 14 < 12$ $-2 < 12$ $3 \times 2 - 2 \times (-14) \geq 21$ $6 + 28 \geq 21$ $34 \geq 21$ The inequalities are both true; therefore, the point is in the required region.

Worked example

Determining the system of linear inequalities for a region

Consider the shaded region shown below.



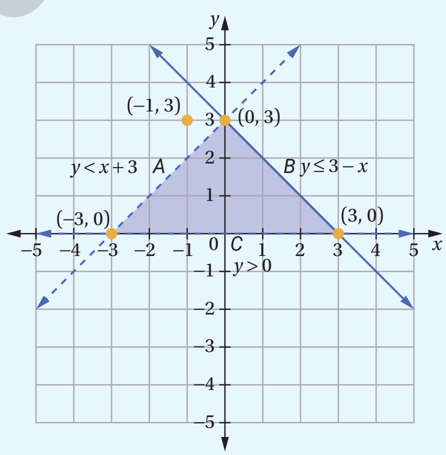
(a) Determine the system of linear inequalities that give the above region.

THINKING	WORKING
Identify the number of boundaries.	There are three boundary lines.
Determine the equation of each boundary line.	Boundary line A: $(x_1, y_1) = (-3, 0)$ and $(x_2, y_2) = (0, 3)$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{0-(-3)} = \frac{3}{3} = 1$ The y-intercept is (0, 3). The equation of boundary line A is $y = x + 3$. Boundary line B: $(x_1, y_1) = (0, 3)$ and $(x_2, y_2) = (3, 0)$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{3-0} = \frac{-3}{3} = -1$ The coordinates of the y-intercept are (0, 3). The equation of boundary line B is $y = 3 - x$ Boundary line C is given by the horizontal line $y = 0$.
Determine the inequality sign corresponding to each boundary line.	The region lies below boundary line A and the boundary line is dashed, meaning it is not included in the region. The inequality is $y < x + 3$. The region lies below boundary line B and the boundary line is solid, meaning it is included in the region. The inequality is $y \le 3 - x$. The region lies above boundary line C and the boundary line is dashed, meaning it is not included in the region. The inequality is $y > 0$.

4

Substitute the x- and y-values of a test point to verify that the inequalities represent the shaded region.	Regarding $(-1, 1)$, $x = -1$ and $y = 1$. Region A: $y < x + 3$ $1 < -1 + 3$ $1 < 2$ Region B: $y \leq 3 - x$ $1 \leq 3 - (-1)$ $1 \leq 2$ Region C: $y > 0$ $1 > 0$ The inequalities are all true and could represent a region containing the test point chosen.

(b) Verify that $(-1, 3)$ is not a solution to the system of linear inequalities.

THINKING	WORKING
Demonstrate graphically that the point does not lie within the required region.	 Graphically the point is outside the region.
Demonstrate algebraically that the point does not lie within the one of the regions.	The graph shows that the point is above boundary line A. Regarding $(-1, 3)$, $x = -1$ and $y = 3$. Region A: $y < x + 3$ $3 \nless -1 + 3$ $3 \nless 2$ The point does not satisfy the inequality for region A; therefore, it is not a solution to the system of linear inequalities.

(c) Determine the area of the shaded region.

THINKING	WORKING
Identify the shape and recall the area formula.	The shaded region is a triangle with a base of 6 units and height of 3 units. Formula for the area of a triangle: $A = \frac{b \times h}{2}$
Calculate the area.	When $b = 6$ and $h = 3$: $A = \frac{b \times h}{2}$ $= \frac{6 \times 3}{2}$ $= 9$
Write the answer.	The area of the shaded region is 9 square units.

Practice

ANSWERS Page XXX

SC 1 I can graph regions in the Cartesian plane using simultaneous inequalities

1 Sketch each pair of inequalities and state the point of the intersection.

(a) $y > 4x$ and $y \geq -4x$

(b) $y \leq 3x$

(c) $y < 28 - 4x$ and $x + y \leq 13$

(d) $y \geq -2x - 1$ and $y < x + 5$

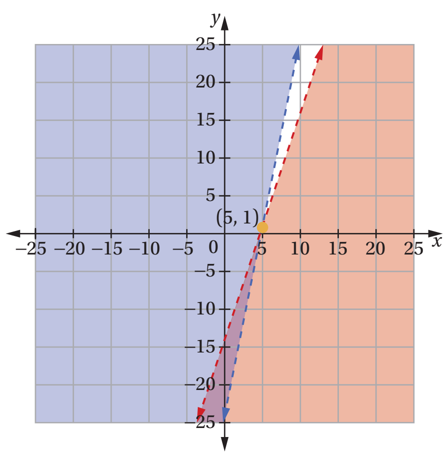
(e) $4x + 3y > 31$ and $2x + 4y < 28$

(f) $x + 2y \geq 6$ and $x - 3y > -12$

2 Consider the following graphs of inequalities:

(a) Determine the system of linear inequalities that would give the above region of intersection.

(b) Demonstrate that (4, –5) is not a solution to the above system of linear inequalities.



3 Sketch $4x + 3y \leq 31$, $x \geq -4$ and $4y + 2x > 28$ onto the same graph.

(a) Determine the coordinates of the points of intersection.

(b) Shade where the three inequalities overlap of intersection.

(c) Calculate the area of the shaded triangle.

4.4 Use linear inequalities to solve real-life problems

Learning intention: To be able to use linear inequalities to solve real-life problems.

Success criteria:

SC 1 I can set up and use linear inequalities to identify all possible solutions for a real-life problem.

4

Lesson warm-up

Making choices

A chocolate bar costs $8 and a bucket of popcorn costs $12. Each student can spend a maximum of $24.

Investigate:

(a) If students only purchase one type of item, then what is the maximum number of items that could be purchased?

(b) If students purchase both types of items, then list all the possible combinations and determine the maximum number of items that could be purchased.

Rather than listing all possible combinations, how can you solve the problems algebraically or graphically?

SC 1 I can set up and use linear inequalities to identify all possible solutions for a real-life problem

Linear inequalities can be used to graphically represent all possible outcomes in real-life problems and help make choices. Represent a real-life problem using a system of linear inequalities as follows.

- Identify the two variables and any restrictions that might apply.
- Write a linear inequality to represent the problem.
- Sketch the graph.
- Identify all possible solutions.

Worked example

Set up and use linear inequalities to identify all possible solutions for a real-life problem

Identify all the combinations of trips to the cinema ($16) and bowling alley ($20) on a holiday budget of no more than $160. Consider how many trips to each you can purchase.

(a) Identify the two variables and any restrictions that might apply.

THINKING	WORKING
Identify the two variables.	Let x be the number of trips to the cinema. Let y be the number of trips to bowling. x and y cannot be negative. $x \geq 0$ $y \geq 0$
Use the variables to write a linear inequality that represents the problem.	The cost of all trips to the cinema and bowling cannot exceed \$160. $16x + 20y \leq 160$

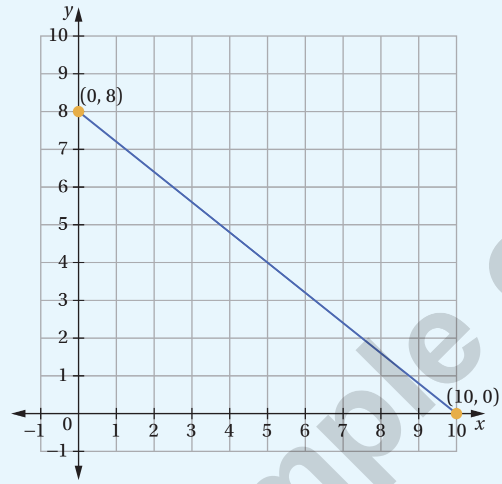
(b) Sketch the graph.

THINKING	WORKING
Determine the coordinates of the intercepts (or two points) of the boundary line.	Boundary line: $16x + 20y = 160$ x-intercept, let $y = 0$: $16x + 20(0) = 160$ $16x = 160$ $x = 10$ The coordinates of the x-intercept are (10, 0). y-intercept, let $x = 0$: $16(0) + 20y = 160$ $20y = 160$ $y = 8$ The coordinates of the y-intercept are (0, 8).
Determine whether the boundary line is to be dashed or solid.	Boundary line: $16x + 20y \leq 160$ The line will be solid because the inequality is $\leq$. Boundary line: $x = 0$ The line will be solid because the inequality is $\geq$. Boundary line: $y = 0$ The line will be solid because the inequality is $\geq$.

4

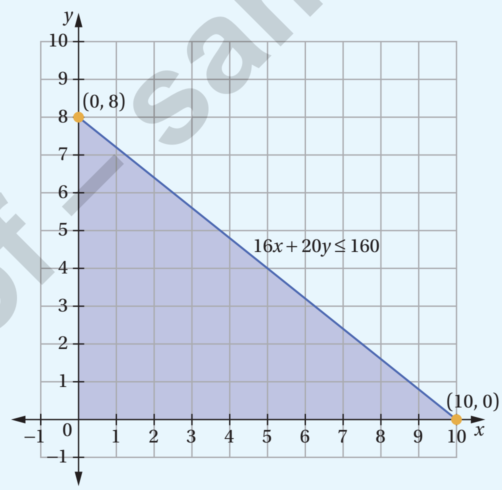
Plot the points on the Cartesian plane and join with the boundary line(s).

Points are (10, 0) and (0, 8).



Shade and label the region.

A region that is 'less than' (<) or 'less than or equal to' (≤) will be shaded below the boundary line. A region that is 'greater than' (>) or 'greater than or equal to' (≥) will be shaded above the boundary line.



Substitute the x- and y-values of a test point to verify that the shaded region is accurate.

(1, 1) at $x = 1$ and $y = 1$

$$16x + 20y \leq 160$$

$$16(1) + 20(1) \leq 160$$

$$36 \leq 160$$

The inequality is true; therefore, the point is in the shaded region.

(c) List any three possible solutions.

THINKING	WORKING
Possible solutions are all points that are inside the shaded region of the graph. List some possible solutions.	Both x and y are non-negative integers; therefore, some of the possible solutions: (1, 7), (2, 6), (3, 5), (4, 4) (5, 3), (6, 3), (7, 2), (8, 1)
List three of the solutions in context.	2 trips to the cinemas, 6 trips to the bowling alley 6 trips to the cinemas, 3 trips to the bowling alley 4 trips to the cinemas, 4 trips to the bowling alley All three options are possible combinations on a budget of $160 or less.

Practice

ANSWERS Page XXX

4

SC 1 I can set up and use linear inequalities to identify all possible solutions for a real-life problem.

1 Sukh has $10 to spend for lunch. Each muffin costs $5 and each cookie costs $2 at his school café. Consider how many of each he can purchase.

- **(a)** Identify the two variables and any restrictions that might apply.
- **(b)** Write a linear inequality that represents the problem.
- **(c)** Sketch the graph.
- **(d)** List any three possible solutions.

2 Christian is a vet who specialises in fish and hamsters. An appointment for fish usually takes 10 minutes and it takes 15 minutes for a hamster. Christian can work a maximum of 7.5 hours a day.

- **(a)** Identify the two variables and any restrictions that might apply.
- **(b)** Write a linear inequality to represent the problem.
- **(c)** Sketch the graph.
- **(d)** Select the possible solutions from the list (45, –1), (10, 30), (10, 12), (30, 0), $\left(30, \frac{32}{3}\right)$

3 Demitri works as a tutor and is paid $3 per hour. He also works as a baby-sitter and is paid $6 per hour. Because of his study, he cannot work more than 15 hours each week, but he needs to make at least $66 to cover his weekly expenses.

- **(a)** Identify the two variables and any restrictions that might apply.
- **(b)** Write inequalities to represent the problem.
- **(c)** Sketch the graph.

4 Students are required to complete both test A and B for a subject. Students need to achieve at least 19 in test A and at least 32 as the combined result to successfully pass the subject.

(a) Identify two variables and any conditions that might apply.

(b) Write the inequities to represent the problem.

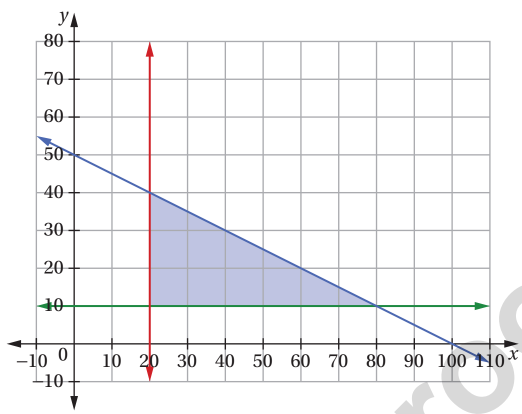
(c) Sketch the graph.

(d) A student achieved 15 in test B. What is the minimum integer result on test A for him to successfully pass this subject?

5 Alexis is planning to sell two handmade crochet items at a local art market: frogs and mini bees. The region below shows information about the number of frogs and the number of mini bees she plans to make in a week.

x = number of frogs she plans to make.

y = number of mini bees she plans to make.



(a) Can Alexis make 40 mini bees and 25 frogs in a week?

(b) Determine the system of inequalities.

(c) The profit for each frog is \$5 and for each mini bee it is \$15. Determine the maximum profit that Alexis can make in that week.

Use inequalities to determine regions involving circles

4.5

Learning intention: To be able to use inequalities to determine regions involving circles.

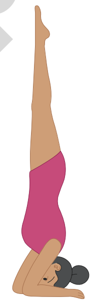
Success criteria:

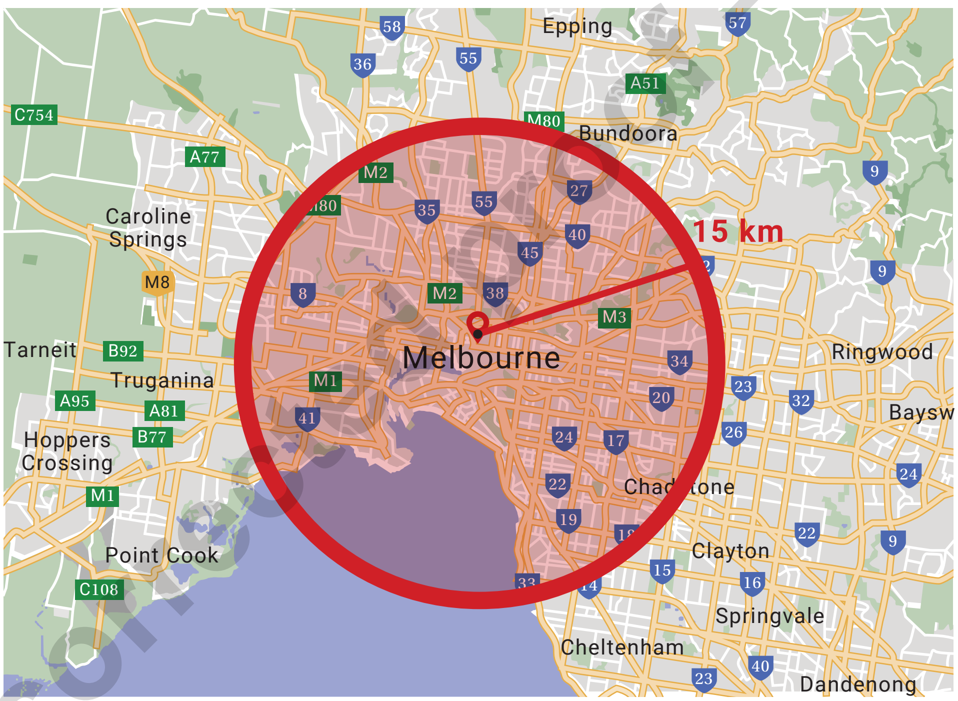
SC 1 I can test whether a given point satisfies an inequality involving a circle.

Lesson warm-up

Location

During the COVID-19 lockdowns, Victorian residents were restricted to travel within a 15 kilometres radius from their home. How can you determine whether a destination is within the 15 kilometres radius, on the exact circle formed by the 15 kilometres radius or farther away from the 15 kilometres radius?
List three locations for each situation using the given graph.

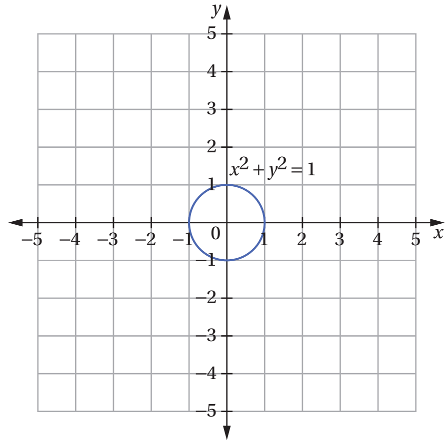




SC 1 I can test whether a given point satisfies an inequality involving a circle

Like linear inequalities, a point can be tested to determine whether it satisfies an inequality involving a circle of a given radius. Substitute the x- and y-values of the point into the inequality to determine which side of the boundary to shade.

The standard equation of a circle is given by $(x-h)^2+(y-k)^2=r^2$, in which h and k are the coordinates of the centre and r is a non-zero real number representing the radius of the circle. For example, the unit circle, with centre (0, 0) and a radius of 1 unit, is defined by $x^2+y^2=1$.



The table below outlines the three alternatives when the x- and y-values of a given point are substituted into a circle inequality.

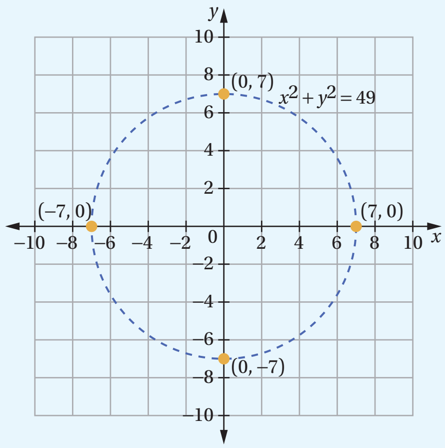
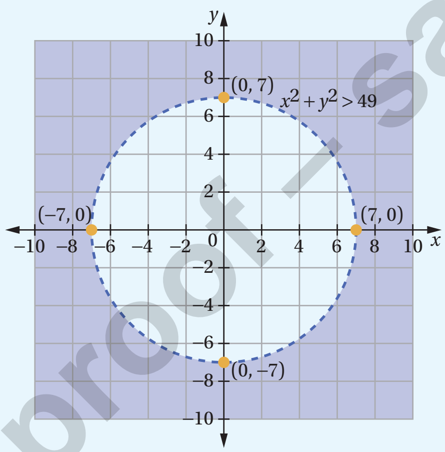
Test inequality	Region of the point
$x^2+y^2<r^2$	inside the circle
$x^2+y^2=r^2$	on the circle
$x^2+y^2>r^2$	outside of the circle

Worked example

Testing whether a given point satisfies an inequality involving a circle

Sketch $x^2+y^2>49$ and determine whether the point (4, 6) lies inside the circle, outside the circle (in the shaded region) or on the circle.

THINKING	WORKING
Determine the coordinates of the intercepts (or two points) of the boundary.	Boundary: $x^2+y^2=49$ x-intercept, let $y=0$: $x^2+0^2=49$ $x^2=49$ $x=\pm7$ The coordinates of the x-intercept are (7, 0) and (−7, 0). y-intercept, let $x=0$: $0^2+y^2=49$ $y^2=49$ $y=\pm7$ The coordinates of the y-intercept are (0, 7) and (0, −7).

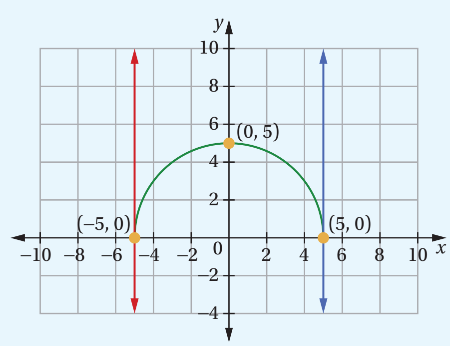
Determine whether the boundary is dashed or solid.	The curve will be dashed because the inequality is $>$.
Plot the points on the Cartesian plane and join with the boundary curve.	
Shade and label the region. Regarding a circle with radius r: $x^2 + y^2 < r^2$ is inside the circle, $x^2 + y^2 = r^2$ is on the circle and $x^2 + y^2 > r^2$ is outside the circle.	
Determine whether the given point (4, 6) lies inside, outside or on the circle.	The test is point (4, 6). Graphically, point (4, 6) lies outside the circle, so it satisfies the inequality. Algebraically, substitute the test point into the inequality: $x^2 + y^2 > 49$ $(4)^2 + (6)^2 > 49$ $52 > 49$ The inequality is true; therefore, the point is outside the circle in the shaded region.

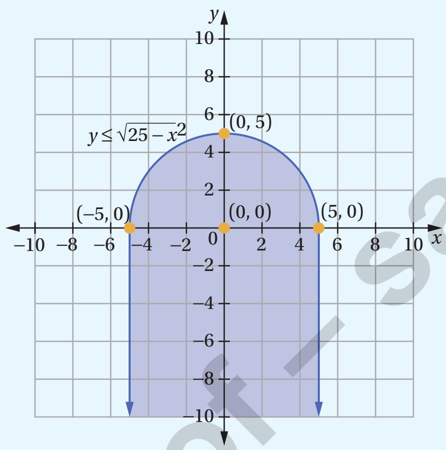
4

Worked example

Use inequalities to determine the boundary of a semicircle

Sketch $y \le \sqrt{25 - x^2}$ and determine of the point (3, 5) lies within the shaded region.

THINKING	WORKING
Determine the intercepts for the boundary. Check to see whether the boundary is included in the region.	The graph of $y = \sqrt{25 - x^2}$ is a semicircle, which is part of the boundary. The inequality $y \le \sqrt{25 - x^2}$ is only defined when the square root is applied to a non-negative number; therefore, $25 - x^2 \ge 0$ $x^2 \le 25$ $-5 \le x \le 5$ The vertical lines $x = -5$ and $x = 5$ are boundary lines when $y \le 0$. Let $y = 0$ to determine the x-intercept. Regarding $y = 0$: $0 = \sqrt{25 - x^2}$ $0^2 = 25 - x^2$ $25 = x^2$ $\pm 5 = x$ The coordinates of the x-intercepts are (5, 0) and (–5, 0). Let $x = 0$ to determine the y-intercept. $y = \sqrt{25 - 0^2}$ $y = \sqrt{25}$ $y = 5$ The coordinates of the y-intercept are (0, 5). The curve will be solid because the inequality is $\le$.
Use the points to sketch the boundary on a Cartesian plane.	Points are (5, 0), (0, 5) and (–5, 0) 

Determine which side of the boundary to shade by testing a point that is not on the boundary line.	The test point is (0, 0). Substitute the test point into the inequality: $y \le \sqrt{25 - x^2}$ $0 \le \sqrt{25 - 0^2}$ $0 \le 5$ This is true. 0 is less than or equal to 5, hence the point satisfies the inequality. The required region is on this side of the boundary line.
Shade the region that includes the point that satisfies the inequality. Label the graph and region.	
Test the inequality to determine whether a given point lies inside, outside or on the boundary.	The test point is (3, 5). Graphically, point (3, 5) lies outside the boundary, so it does not satisfy the inequality. Algebraically, substitute the test point into the inequality: $5 \le \sqrt{25 - 3^2}$ $5 \le \sqrt{16}$ $5 \not\le 4$ This is false. 5 is not less than or equal to 4, hence the point does not satisfy the inequality. The given point is on the other side of the boundary line.

Practice

ANSWERS Page XXX

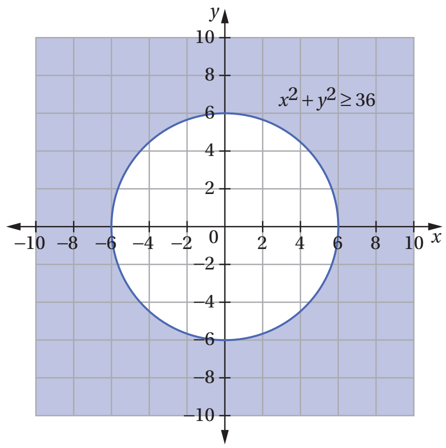
SC 1 I can test whether a given point satisfies an inequality involving a circle

1 Graph the following regions.

(a) $x^2 + y^2 = 4$ **(b)** $x^2 + y^2 \ge 9$ **(c)** $x^2 + y^2 < 16$

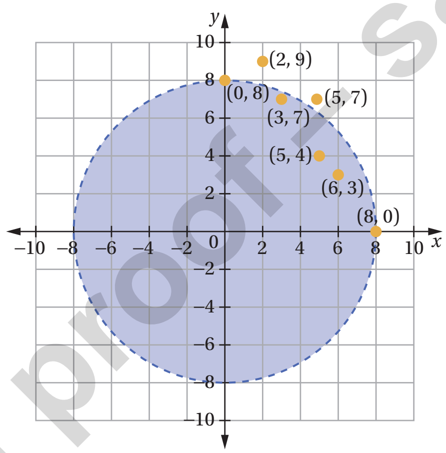
(d) $x^2 + y^2 \le 25$ **(e)** $x^2 + y^2 > 49$

2 Decide whether the following points lie inside the shaded region, outside the shaded region or on the boundary.



(a) (–4, 4) **(b)** (2, 6) **(c)** (0, 6)

3 For the following graph, write the coordinates of 4 points that lie:



(a) on the boundary $x^2 + y^2 = 64$ (note that the boundary is not included in the shaded region)

(b) inside the shaded region $x^2 + y^2 < 64$

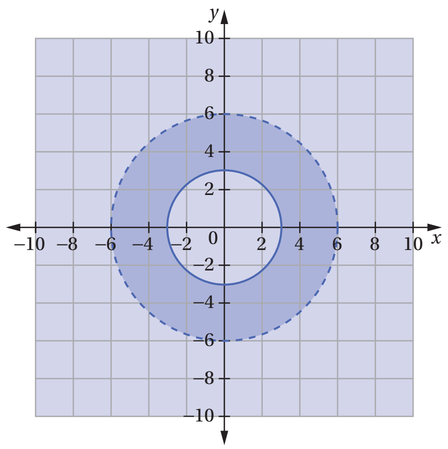
(c) outside the shaded region $x^2 + y^2 > 64$

4 Consider $y > \sqrt{16 - x^2}$.

(a) Sketch the inequality on a Cartesian plane.

(b) Select the points that belong to the inequality from the list: (2, 1), (1, –1), (–1, 5), (5, 1), (0, 4), (–3, 4)

5 Consider the following graph.



(a) Determine the inequalities that best represent the graph shown.

(b) Decide whether the following points lie inside, outside or on the boundary.

(−4, 2), (−2, 1), (−6, 0), (0, 3)

6 Consider $y \geq \sqrt{81 - x^2}$.

(a) State the boundary.

(b) Sketch the inequality.

(c) Decide whether (−3, 5) lies inside, outside or on the boundary.

7 Consider $x^2 + y^2 \leq 16$ and $y > x$.

(a) State the boundaries.

(b) Sketch the inequalities.

(c) State the points of intersection. Give your answer as exact values.

8 Determine whether (3, 4) satisfies the following inequalities.

(a) $y^2 > 64 - x^2$

(b) $y \leq \sqrt{64 - x^2}$

(c) $y^2 < 3x$

(d) $(x - 2)^2 + (y - 2)^2 < 64$

9 Consider $y^2 < 36 - x^2$.

(a) Sketch the inequality.

(b) Select the points that belong to the inequality from the list: (1, −3), (5, −5), (−1, −5), (−7, 3), $\left(4, \frac{7}{3}\right)$